

1306/311
1307/311
1308/311
MATHEMATICS
June/July 2022
Time: 3 hours



THE KENYA NATIONAL EXAMINATIONS COUNCIL
CRAFT CERTIFICATE IN CARTOGRAPHY
CRAFT CERTIFICATE IN PHOTOGRAMMETRY
CRAFT CERTIFICATE IN LAND SURVEYING

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet;

Mathematical tables/Scientific calculator;

This paper consists of EIGHT questions.

Answer any FIVE questions.

All questions carry equal marks.

Maximum marks for each part of a question are as indicated.

Candidates should answer the questions in English.

This paper consists of 6 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. ✓ (a) The sum of the 4th and 7th terms of an arithmetic progression is 46, while the sum of the 5th and 8th terms is 58.
Determine the:

- (i) first term
(ii) common difference

Handwritten work for question 1(a):
 $2a + 9d = 46$
 $2a + 11d = 58$
 $-2d = -12$
 $d = 6$
 $2a + 9(6) = 46$
 $2a + 54 = 46$
 $2a = 46 - 54$
 $2a = -8$
 $a = -4$

(9 marks)

- (b) Determine the sum to infinity of the series

$$30 + 10 + 3\frac{1}{3} + \dots$$

(5 marks)

- (c) A survey export and import company in Kenya owes a firm in the United States US\$55600. The company can pay this debt in Kenya shillings (Ksh) or in sterling pounds (£) through their account in Britain.

The bank in Kenya charges a commission of 5% while the British bank charges 3% commission on the transaction.

The currency exchange rates are

- 1 US\$ = Ksh.113
 1 sterling £ = US \$ 1.32
 1 sterling £ = Ksh.150

Determine the difference between the costs of the two methods of transaction.

(6 marks)

2. (a) Given the matrices

$$A = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \text{ and } B = \begin{pmatrix} -2 & 3 \\ 8 & 4 \end{pmatrix}$$

Determine:

- (i) $(A+B)^T$;
 (ii) $A - B^T$;
 (iii) $(AB)^T$.

Handwritten work for question 2(a):
 $(A+B)^T = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} + \begin{pmatrix} -2 & 3 \\ 8 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ 13 & 11 \end{pmatrix}$
 $A - B^T = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} - \begin{pmatrix} -2 & 8 \\ 3 & 4 \end{pmatrix} = \begin{pmatrix} 4 & -5 \\ 2 & 3 \end{pmatrix}$
 $(AB)^T = \begin{pmatrix} 2 & 3 \\ 5 & 7 \end{pmatrix} \begin{pmatrix} -2 & 3 \\ 8 & 4 \end{pmatrix} = \begin{pmatrix} 20 & 18 \\ 46 & 43 \end{pmatrix}$

(10 marks)

(b) Given the matrix $M = \begin{bmatrix} 4 & 6 \\ 3 & -8 \end{bmatrix}$ and $N = \begin{bmatrix} -8 & -6 \\ 3 & 4 \end{bmatrix}$

(i) Show that $MN = \lambda I$ where I is an identity matrix.

(ii) Determine M^{-1} and hence solve the equations..

$$4x + 6y = 62$$

$$3x - 8y = -41$$

Handwritten solution for (ii):

$$\begin{pmatrix} 4 & 6 \\ 3 & -8 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 62 \\ -41 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 6 \\ 3 & -8 \end{pmatrix}^{-1} \begin{pmatrix} 62 \\ -41 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 4 & 6 \\ 3 & -8 \end{pmatrix}^{-1} = \frac{1}{-32-18} \begin{pmatrix} -8 & -6 \\ -3 & 4 \end{pmatrix} = \frac{1}{-50} \begin{pmatrix} -8 & -6 \\ -3 & 4 \end{pmatrix}$$

$$\begin{pmatrix} 8/50 & 6/50 \\ 3/50 & -4/50 \end{pmatrix} \begin{pmatrix} 62 \\ -41 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 8/50(62) + 6/50(-41) \\ 3/50(62) - 4/50(-41) \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 49.6 - 49.2 \\ 37.2 + 32.8 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$\begin{pmatrix} 0.4 \\ 70 \end{pmatrix} = \begin{pmatrix} x \\ y \end{pmatrix}$$

$$x = 0.4, y = 70$$

3. (a) A land survey headquarters building is supplied with water by two pumps P and Q. The probability that pump P fails is $\frac{1}{8}$ and the probability that pump Q fails is $\frac{1}{4}$. Calculate the probability that:

- (i) both pumps are working.
- (ii) both pumps fail.
- (iii) only one pump is working.

(7 marks)

(b) Table 1 shows the marks scored by 120 certificate in survey students.

Table 1

Marks	0 - 9	10 - 19	20 - 29	30 - 39	40 - 49	50 - 59	60 - 69	70 - 79	80 - 89
Number of students	22	16	12	9	10	x	y	8	3

If the mode mark is $\frac{873}{14}$, determine the

- (i) values of x and y;
- (ii) mean;
- (iii) median.

(13 marks)

4. (a) Use Simpson's rule with 7 ordinates to determine the area bounded by the curve.
 $y = \frac{1}{\sqrt{2 + \cos x}}$ for values of x between 0 and $\frac{\pi}{2}$.

(10 marks)

- (b) Figure 1 shows a solid frustrum of a right circular cone. The frustrum has a cylindrical hole of diameter 5 cm drilled through its. The height of the frustrum is 15 cm. The radius of the cone of the top is 10 cm.

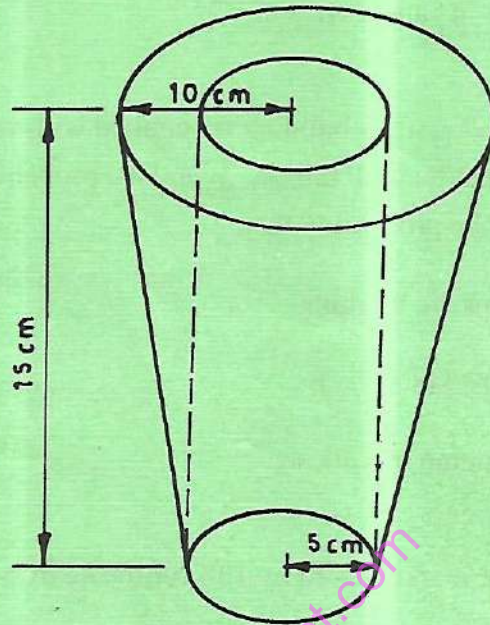


Fig.1

Calculate the volume of the frustrum.

(10 marks)

5. (a) Given that $\cos \theta = 0.6$, determine the other 5 trigonometric ratios of θ . (6 marks)
- (b) Prove the trigonometric identity

$$\frac{1}{\cos \theta} - \frac{\cos \theta}{1 + \sin \theta} = \tan \theta$$

(4 marks)

- (c) Solve the equation:

$$\cos 2\theta + 5 \sin \theta = 3, \quad 0^\circ \leq \theta \leq 360^\circ$$

(10 marks)

6. (a) XY and Z three quantities such that X varies directly as the square of Y and inversely as the square root of Z .

Given that $X = 18$ when $Y = 3$ and $Z = 4$, calculate the value of X when $Y = 6$ and $Z = 4$.

(8 marks)

- (b) Figure 2 shows a parallelogram $OACB$ in which M is the mid point of AC and OM produced meets BC also produced at X .

Given that $\vec{OA} = \underline{a}$ and $\vec{OB} = \underline{b}$

determine the:

- (i) Values of r and s such that $\vec{OA} = r\vec{OM}$ and $\vec{CX} = s\vec{BC}$.
- (ii) ratio $BC : BX$

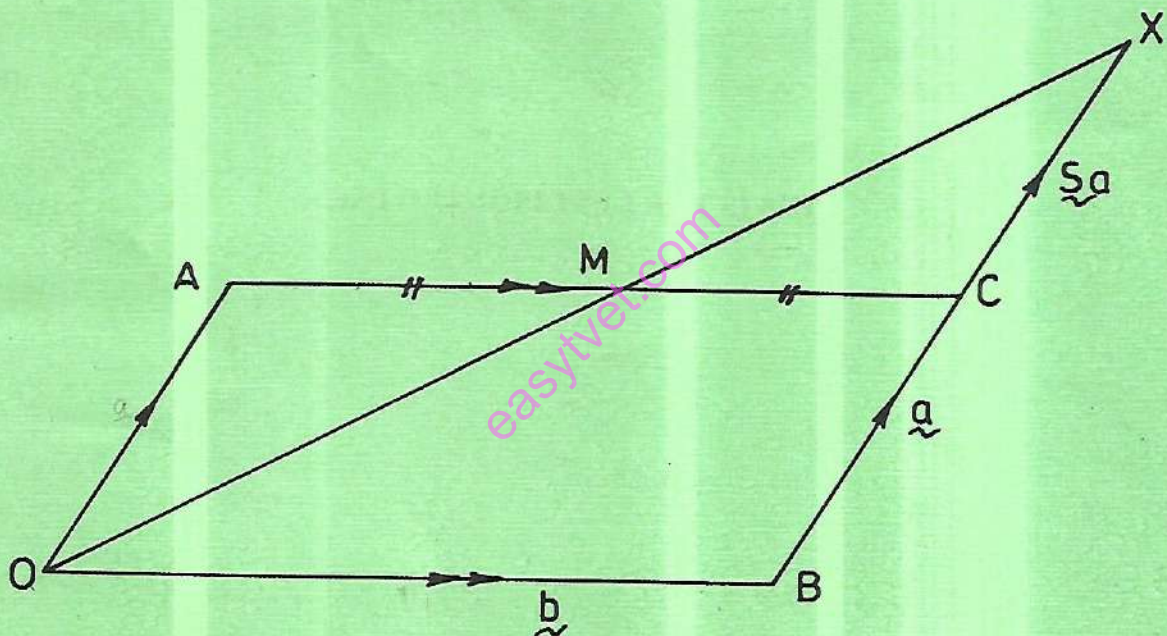


Fig.2

(12 marks)

7. (a) Without using a calculator or mathematical tables, simplify the expression:

$$\frac{625^{\frac{3}{4}} \times 4^{\frac{3}{2}} \times 0.42}{12^{\frac{1}{2}} \times 3^{\frac{3}{2}}}$$

leaving the answer as a mixed number.

(9 marks)

(b) Solve the equation:

$$9^x + 3^{2x} - 1 = 53$$

(6 marks)

(c) Use the method of completing the square to solve the equation.

$$2x^2 + 5x + 7 = 0$$

(5 marks)

8. (a) Use elimination method to solve the equations

$$3x + 2y + 5z = 20$$

$$4x + 5y - 7z = -35$$

$$5x - 8y + 2z = 42$$

(12 marks)

(b) Use your graph to solve the equation

$$2x^2 + 2x = 6$$

(8 marks)

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