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— Exam

2405/301
MATHEMATICS
Oct./ Nov. 2018
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN APPLIED STATISTICS

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
- *Answer booklet;*
- *Mathematical tables;*
- *Non-programmable scientific calculator;*
- *Drawing instruments.*

This paper consists of ***EIGHT*** *questions.*
Answer any ***FIVE*** *questions in the answer booklet provided.*
Maximum marks for each part of a question are indicated.
Candidates should answer the questions in English.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Given that $Z_1 = 2 - j$

$$Z_2 = 1 + 2j \text{ and}$$

$$\frac{2}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2},$$

determine Z in the form a + bj. (8 marks)

(b) Determine the cube roots of $Z = 5 + 12j$, giving the answers in the form $a + bj$, where a and b are real constants correct to 3 decimal places. (12 marks)

2. (a) (i) Determine the Maclaurin's series expansion of $ln(1 - x)$ in ascending powers of x up to the term in x^5.

(ii) Hence determine the value of $ln(0.98)$ correct to four decimal places. (11 marks)

(b) (i) Use Taylor's theorem to expand $\sin\left(\frac{\pi}{4} + h\right)$ in ascending powers of h upto the term in h^3.

(ii) Hence determine the value of sin 46° correct to 5 decimal places. (9 marks)

3. (a) Given that $Z = 3x^2y^2 + 6x^2 + y^3 + 6x^3y^2 + 7x$, determine, at the point (2,1), the values of:

(i) $\frac{\partial^2 Z}{\partial x^2}$

(ii) $\frac{\partial^2 Z}{dy dx}$ (6 marks)

(b) The radius of a cone is increasing at the rate of 0.5 cm/s and the height is decreasing at the rate of 0.8 cm/s. Determine the rate at which the volume of the cone is changing at the instant when the radius is 8 cm and the height is 10 cm. (7 marks)

(c) Determine the stationary points of the function $f(x,y) = x^2 + y^3 - 4xy$. (7 marks)

4. (a) Solve the equation

$$\begin{vmatrix} x & 5 \\ -9 & x+3 \end{vmatrix} = \begin{vmatrix} 5 & 2 \\ -5 & 3x \end{vmatrix}$$

(5 marks)

(b) (i) Determine the inverse of the matrix $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 6 & 8 & 9 \end{bmatrix}$.

(ii) Use the result obtained in b(i) to solve the following linear simultaneous equations:

$$x + 2y + 3z = -2$$
$$4x + 5y + 7z = -9$$
$$6x + 8y + 9z = -11$$

(15 marks)

5. (a) Evaluate the following double integrals:

(i) $\int_{-1}^{0}\int_{-1}^{y+1}(xy - x)\,dx\,dy$;

(ii) $\int_{1}^{2}\int_{x}^{4} x^2 y\,dy\,dx$.

(11 marks)

(b) Determine the volume of the solid lying below the plane $Z = 8 - x - y$ and above the triangular region R, bounded by $x = 0, y = 0$ and $y = 4 - 2x$. (9 marks)

6. (a) Show that the solution of the differential equation $y^2 + (xy + x^2)\frac{dy}{dx} = 0$ is given by $xy^2 = k(x + 2y)$, where k is a constant. (14 marks)

(b) The temperature, y degrees, of a body, t minutes after being placed in a certain room, satisfies the differential equation $6\frac{d^2y}{dt^2} + \frac{dy}{dt} = 0$. If at $t = 0, y = 20$ and $\frac{dy}{dt} = -2$, determine an expression for temperature y in terms of t. (6 marks)

7. (a) (i) Show that one root of the equation $x^2 + 2x - 5 = 0$ lies between 1 and 2.

(ii) Use Newton-Raphson method to determine the root, taking the first approximation $x_0 = 1.5$. Give the answer correct to four decimal places.

(14 marks)

(b) Table 1 satisfies a function f(x).

Table 1

x	1	2	3	4	5	6	7
f(x)	10	19	40	79	142	235	364

Use Newton-Gregory backward difference interpolation formula to determine the value of f(7.4). (6 marks)

8. (a) Use Simpson's rule with seven ordinates to evaluate the integral $\int_{0}^{1.2} \sqrt{1+x^2}\,dx$, giving the answer correct to four decimal places. (10 marks)

(b) Given the vectors $\underset{\sim}{u} = \begin{bmatrix} 4 \\ -3 \\ 7 \end{bmatrix}$ and $\underset{\sim}{v} = \begin{bmatrix} 2 \\ -5 \\ -4 \end{bmatrix}$. Determine:

(i) $\underset{\sim}{W} = 2\underset{\sim}{u} + 3\underset{\sim}{v}$;

(ii) $|\underset{\sim}{W}|$

(5 marks)

(c) Figure 1 shows a triangle OAB, with P and Q on OA and OB respectively, such that $\overrightarrow{OP} = \frac{1}{3}\overrightarrow{OA}, \overrightarrow{OQ} = \frac{1}{3}\overrightarrow{OB}, \overrightarrow{PR} = \frac{1}{4}\overrightarrow{PB}$. If $\overrightarrow{OA} = 12\underset{\sim}{a}, \overrightarrow{OB} = 12\underset{\sim}{b}$, determine in terms of $\underset{\sim}{a}$ and $\underset{\sim}{b}$, vectors:

(i) $\overrightarrow{AB}$;

(ii) $\overrightarrow{AR}$.

(5 marks)

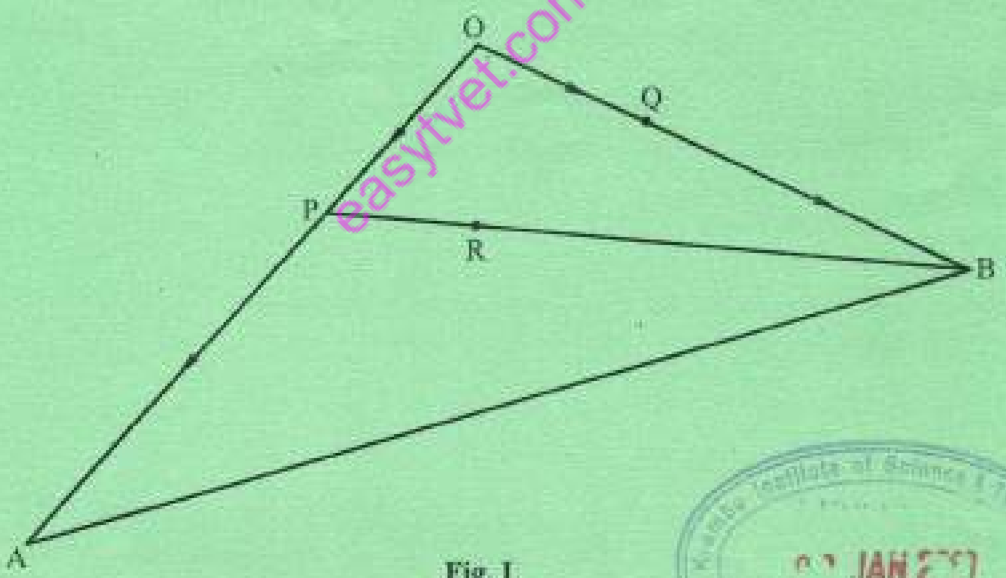

Fig. 1

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