2305/301
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MATHEMATICS
Oct./Nov. 2009
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN BUILDING
DIPLOMA IN QUANTITY SURVEYING
DIPLOMA IN CIVIL ENGINEERING
DIPLOMA IN HIGHWAY ENGINEERING
DIPLOMA IN ARCHITECTURE

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:

Answer booklet
Mathematical tables/calculator
Drawing instruments.

*Answer any **FIVE** of the **EIGHT** questions in this paper.*
ALL questions carry equal marks.
Maximum marks for each part of a question are as shown.

This paper consists of 4 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

Turn over

1. (a) Given that $8\cos\theta + 25\sin\theta \equiv R\cos(\theta - \alpha)$ where $R > 0$ and $0^\circ \leq \alpha \leq 90^\circ$, find the value of R and α and hence solve the equation:

$8\cos\theta + 25\sin\theta = 2$ for $0^\circ \leq \theta \leq 360$

(9 marks)

(b) A person on a boat at point P observes the top of a hill at an angle of elevation of 20^0. When the boat moves 0.5 km from P away from the hill to a point Q, the angle of elevation is observed to be 15°.

Calculate:
(i) the distance from P to the bottom of the hill;
(ii) the height of the hill;
(iii) the distance in metres from P to the top of the hill. (8 marks)

(c) Calculate the area of a lawn with sides 58m, 52m and 28m. (3 marks)

2. (a) Given the complex number $z = -2.598 + 1.5j$ express z in the form $r(\cos\theta + j\sin\theta)$. Hence find Z^4.

(7 marks)

(b) Find $\sqrt{(2 - 7j)}$ and give the answer in the form a + jb. (10 marks)

(c) Determine the angle between the two vectors $\underset{\sim}{V} = 3\underset{\sim}{i} + 5\underset{\sim}{j}$ and $\underset{\sim}{U} = \underset{\sim}{i} + 4\underset{\sim}{j}$. (3 marks)

3. (a) Find $\int \frac{x\,dx}{(x-2)(x^2+3)}$.

(9 marks)

(b) Calculate the centroid of area bounded by the curve $y = e^{2x}$ and the x - axis between x = 0 and x = 2. (11 marks)

4. Given the matrices:

$$A = \begin{pmatrix} 3 & 0 & 1 \\ 2 & 2 & 1 \\ 2 & 1 & 1 \end{pmatrix} \quad B = \begin{pmatrix} 0 & 3 & 2 \\ 1 & 2 & 2 \\ 2 & 1 & 3 \end{pmatrix} \quad \text{and} \quad C = \begin{pmatrix} 4 & 2 & 4 \\ 9 & 7 & 6 \\ 4 & 5 & 7 \end{pmatrix}.$$

(a) Determine:

(i) $M = AB + C$;

(ii) M^{-1}. (12 marks)

(b) Using the results in (a) above solve the simultaneous equations:

$$6x + 12y + 13z = 20$$
$$13x + 18y + 17z = 29$$
$$9x + 15y + 19z = 32$$

(5 marks)

(c) Show that $(AB)^T = B^T A^T$. (3 marks)

5. Solve the following differential equations:

(a) $4x^2 \frac{dy}{dx} = 3x^2 + y^2$. (10 marks)

(b) $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 3 = 0$ given that $x = 0$, and $\frac{dx}{dt} = 3$ at $t = 0$.

(10 marks)

6. (a) $\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 3 = 0$.

Given that $f(x,y) = \tan^{-1}(^y/_x)$, show that $\frac{\partial^2 f}{\partial x \partial y} = \frac{\partial^2 f}{\partial y \partial x}$

(8 marks)

(b) The volume of a cone is given by $V = \frac{1}{3}\pi r^2 h$.
Find the approximate change in volume if the radius r increases by 0.02 and the height decreases by 0.03 at the time when radius is 5cm and height is 7cm. (5 marks)

(c) The surface area S of a cone of base radius r and height h is given by $s = \pi r^2 + \pi r\sqrt{r^2 + h^2}$. Calculate the rate at which the surface area is changing when $h = 7$cm and $r = 4$cm, if h is increasing at the rate of 0.4cm/s while r is decreasing at the rate of 0.7cm/s. (7 marks)

7. (a) Sketch the graph of the curve $y = 8x^3 - 24x + 16$ given that at $y = 0$, $x = 1$.

(14 marks)

(b) Find the volume generated when the area between the curve in (a) above, the x - axis and the y-axis is rotated about the x- axis through 360^0. (6 marks)

8. (a) A department has four lecture theatres. The probability that any one lecture theatre is unoccupied is $\frac{1}{7}$. Find the probability that:

(i) any three of the four lecture theatres are unoccupied;

(ii) all the lecture theatres are occupied. (6 marks)

(b) A continuous random variable t has a probability density function defined by:

$$f(t) = \begin{cases} k(1-t)^2 & 1 \leq + \leq 4 \\ 0 & \text{elswhere} \end{cases}$$

Find (i) the value of the constant k;

(ii) the mean and standard deviation;

(iii) the probability that t lies between 1.5 and 2.5. (14 marks)