2310/301
2311/301
2312/301
2313/301
MATHEMATICS
Oct./Nov. 2022
Time: 3 hours

THE KENYA NATIONAL EXAMINATIONS COUNCIL

DIPLOMA IN PHOTOGRAMMETRY AND REMOTE SENSING
DIPLOMA IN CARTOGRAPHY
DIPLOMA IN LAND SURVEYING
DIPLOMA IN MAP REPRODUCTION

MATHEMATICS

3 hours

INSTRUCTIONS TO CANDIDATES

You should have the following for this examination:
Answer booklet;
Mathematical tables/ Scientific calculator.
*Answer **FIVE** of the following **EIGHT** questions.*
All questions carry equal marks.
Maximum marks for each part of a question are as indicated.
Candidates should answer the questions in English.

This paper consists of 5 printed pages.

Candidates should check the question paper to ascertain that all the pages are printed as indicated and that no questions are missing.

1. (a) Solve the equation $\begin{vmatrix} 1-\lambda & 2 & 2 \\ 2 & 1-\lambda & -2 \\ 2 & -2 & 1-\lambda \end{vmatrix} = 0$ (8 marks)

(b) Use Simpson's rule with 8 strips to evaluate the integral

$$\int_0^{\pi} \frac{2d\theta}{\sqrt[3]{5+\frac{4}{3}\sin^3\theta}}$$ (12 marks)

2. (a) If $z = x^2 \tan^{-1}\left(\frac{y}{x}\right) - y^2 \tan^{-1}\left(\frac{x}{y}\right)$, prove that

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$$ (6 marks

(b) The deflection at the centre of a rod of length l and diameter d supported at its ends and loaded at the centre with weight W is given by:

$$D = \frac{kWl^3}{d^4}$$

Use partial differentiation to determine the percentage change in D if W, l and d increase by 4%, 5% and 3% respectively. (6 marks)

(c) Locate the stationary points of the function

$f(x, y) = 2x^2 + 7xy + 5y^2 - 40x - 61y + 30$, and determine their nature. (8 marks)

3. (a) If air temperature is maintained at 30°C and a body cools down from 80°C to 60°C in 12 minutes, determine the temperature of the body after 24 minutes. (8 marks)

(b) Solve the differential equation:

$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = \sin 2t$ given that when $t = 0, x = 3$ and $\frac{dx}{dt} = 0$. (12 marks)

4. (a) Solve the equation:

$z\bar{z} - 3\bar{z} + 5j = 5 + 8$, given that $z = x + yj$ (6 marks)

(b) (i) Use Demoivres theorem to expand $\cos 4\theta$ and $\sin 4\theta$ in terms of powers of cosines and sines of θ.

(ii) Hence obtain an expression for $\tan 4\theta$ in terms of $\tan\theta$.

(iii) Use the result in (ii) above to solve the equation:

$$t^4 + 4t^3 - 6t^2 - 4t + 1 = 0.$$

(14 marks)

5. (a) A continuous random variable T has a probability density function defined by:

$$f(t) = \begin{cases} 4c^2 e^{-2ct}; & t > 0 \\ 0, & elsewhere \end{cases}$$

where c is a constant.

Determine the:

(i) value of c;
(ii) median;
(iii) interquartile range.

(14 marks)

(b) **Table 1** represents the marks scored by 30 Diploma in Land Surveying students in a Cartography examination.

Table 1

Marks	0-5	5-10	10-15	15-20	20-25	25-30	30-35	35-40
No. of students	2	3	4	x	y	3	4	2

Determine the values of x and y if the mode mark is 18.

(6 marks)

6. (a) Use Newton-Raphson method to solve the equation:

$x^6 - 3x^4 + 2x^2 - 3 = 0$ near $x = 2.2$.

(10 marks)

(b) Use the trapezoidal rule with seven ordinates to evaluate the integral:

$$\int_0^{\pi} \frac{3d\theta}{\sqrt[3]{4 + \frac{1}{7}\sin^2\theta}}$$

(10 marks)

7. (a) Evaluate the integrals

(i) $\int \frac{4\sin x + 5\cos x}{8\sin x + 3\cos x} dx$

(ii) $\int_0^1 \frac{4x^2 + 7x + 3}{(x+3)(x^2+9)}$

(13 marks)

(b) Determine the surface area of the solid formed by revolving the cordioid

$r = 2(1+\cos\theta)$ between) and π about the polar axis. (7 marks)

8. (a) Prove the identity:

$$\frac{1+\sin\theta-\cos\theta}{1+\sin\theta+\cos\theta}=\frac{1}{\text{cosec}\,\theta+\cos\theta}$$ (6 marks)

(b) Show that the polar equation of the curve $x^2+24y-64=0$ is given by

$$r=\frac{12}{1+\sin\theta}$$ (7 marks)

(c) Solve the equation:

$6\sin x+8\cos x=5, 0^\circ \le x \le 360^\circ$ (7 marks)

(1b)

0	3.142	6.283	9.425	12.566	15.708	19.350	21.991
0.447	0.447	0.447	0.397	0.435	0.446	0.445	0.444
y_0	y_1	y_2	y_3	y_4	y_5	y_6	y_7

$h = \frac{7\pi - 0}{7} = 3.142$

$\frac{3.142}{3}\left[(y_0+y_7)+4(\text{odds})+2(\text{evens})\right]$

$\frac{3.142}{3}\left[(0.447+0.444)+4(0.447+0.397\{0.446)+2(0.447+0.435+0.445)\right]$

$= 9.114135$