CHAPTER 2: ENGINEERING MATHEMATICS /APPLY ENGINEERING MATHEMATICS

2.1. Introduction of the Unit of Learning / Unit of Competency

This unit describes the competencies required by a technician in order to apply algebra, apply trigonometry and hyperbolic functions, apply complex numbers, apply coordinate geometry, carry out binomial expansion, apply calculus, solve ordinary differential equations, solve Laplace transforms ,apply power series, apply statistics, apply numerical methods, apply vector theory apply matrix, apply Fourier series and numerical methods.

2.2. Performance Standard

The trainee will apply algebra, trigonometry and hyperbolic functions, complex numbers, coordinate geometry, carryout binomial expansion, calculus ordinary differential equations, Laplace transforms, power series, statistics Fourier series, vector theory, matrix and numerical methods in solving engineering problems.

2.3. Learning Outcomes

2.3.1. List of Learning Outcomes

a) Apply Algebra
b) Apply Trigonometry and hyperbolic functions
c) Apply complex numbers
d) Apply coordinate geometry
e) Carry out Binomial Expansion
f) Apply Calculus
g) Solve ordinary differential equations
h) Apply Laplace transforms
i) Apply power series
j) Apply statistics
k) Apply Fourier series
l) Apply vector theory
m) Apply matrix
n) Numerical methods

2.3.2. Learning Outcome No. 1. Apply Algebra

2.3.2.1 Learning Activities.

Learning Outcome No. 1. Apply Algebra	
Learning Activities	Special Instructions
• Perform calculations involving indices as per the concept • Perform calculations involving logarithms as per the concept • Solving simultaneous as per the rules • Solving quadratic equations as per the concept	

2.3.2.2. Information Sheet No.2/ LO1.

Algebra is used throughout engineering, but it is most commonly used in mechanical, electrical, and civil branches due to the variety of obstacles they face. Engineers need to find dimensions, slopes, and ways to efficiently create any structure or object.

Definitions

Algebra is the study of mathematical symbols and the rules for manipulating these symbols; it is a unifying thread of almost all of mathematics. It includes everything from elementary equation solving to the study of abstractions such as groups, rings, and fields.

Content

Indices

An index number is a number which is raised to a power. The power, also known as the index, tells you how many times you have to multiply the number by itself. For example, 2^5 means that you have to multiply 2 by itself five times = 2×2×2×2×2 = 32

Laws of indices

(i) $x^0 = 1$

(ii) $x^{-n} = \frac{1}{x^n}$

(iii) $x^n . x^m = x^{n+m}$

(iv) $x^n - x^m = x^{n-m}$

(v) $(x^n)^m = x^{m.n}$

(vi) $x^{\frac{n}{m}} = \sqrt[m]{x^n}$

Application of rules of indices in solving problems.

- $y^a \times y^b = y^{a+b}$

Examples

$2^4 \times 2^8 = 2^{12}$

$5^4 \times 5^{-2} = 5^2$

- $y^a \div y^b = y^{a-b}$

Examples

$2-3 = 1/23 = 1/8$

$3^{-1}=1/3$

- $ym/n = (n\sqrt{y})m$

Examples

$16^{1/2} = \sqrt{16} = 4$

$8^{2/3} = (^3\sqrt{8})^2 = 4$

- $(y^n)^m = y^{nm}$

Example

$2^5 + 8^4$

$= 2^5 + (2^3)^4$

$= 2^5 + 2^{12}$

$y^0 = 1$

Example

$5^0 = 1$

Logarithms

If a is a positive real number other than 1, then the logarithm of x with base a is defined by:

$$y = \log_a x \quad \text{or} \quad x = a^y$$

Laws of logarithms

(i) $log_a(xy) = log_a x + log_a y$

(ii) $log_a\left(\frac{x}{y}\right) = log_a x + log_a y$

(iii) $log_a(x^n) = nlog_a x$ for every real number

Simultaneous equations with three unknown

Simultaneous equations are equations which have to be solved together to find the unique values of the unknown quantities which are time for each of the equations. Two methods of solving simultaneous equations analytically are:

(i) By substitution

(ii) By elimination

Examples

Solve the following simultaneous equation by substitution methods

$$3x - 2y + = 1 \dots\dots\dots.. \text{(i)}$$
$$x - 3y + 2z = 13 \dots\dots. (ii)$$
$$4x - 2y + 3z = 17 \dots\dots\dots (iii)$$

From equation (ii) $x = 13 + 3y - 2z$

Substituting these expression $(13 + 3y - 2z \ \ for\ x\ \ gives)$

$$3(13 + 3y - 2z) + 2y + z = 1$$
$$39 + 9y - 6z + 2y + x = 1$$
$$11y - 5z = -38 \dots\dots\dots\dots\dots.. (iv)$$
$$4(13 + 3y - 2z) + 3z - 2y = 17$$
$$52 + 12y - 8z + 3z - 2y = 17$$
$$10y - 5z = -35 \dots.. (v)$$

Solve equation (iv) and (v) in the usual way,

From equations (iv) $5z = 11y + 38; z = \frac{11y+38}{5}$

Substituting this in equation (v) gives

$$10y - 5\left(\frac{11y + 38}{5}\right) = -35$$
$$10y - 11y - 38 = -35$$
$$-y = -35 + 38 = 3$$
$$y = -3$$
$$z = \frac{11y + 38}{5} = \frac{-33 + 38}{5} = \frac{5}{5} = 1$$

But $x = 13 + 3y - 2z$

$x = 13 + 3(-3) - 2(1)$

$=13 - 9 - 2$

$=2$

Therefore, $x = 2,\ y = -3\ \ and\ z = 1$ is the required solution

For more worked examples on substitution and elimination method refer to Engineering Mathematics by A.K Stroud.

Quadratic Equations

Quadratic equation is one in which the highest power of the unknown quantity is 2. For example $2x^2 - 3x - 5 = 0$ is a quadratic equation? The general form of a quadratic equation is $ax^2 + bx + c = 0, where\ a, b\ and\ c$ are constants and $a \neq 0\ of\ solving\ quadratic\ equations$.

a. By factorization (where possible)
b. By completing the square
c. By using quadratic formula
d. Graphically

Example

Solve the quadratic equation $x^2 - 4x + 4 = 0$ by factorization method

Solution

$$x^2 - 4x + 4 = 0$$
$$x^2 - 2x - 2x + 4 = 0$$
$$x(x-2) - 2(x-2) = 0$$
$$(x-2)(x-2) = 0$$

i.e $x - 2 = 0\ \ or\ x - 2 = 0$

$x = 2\ \ or\ x = 2$

i.e. the solution is $x = 2$(twice)

For more worked examples on how to solve quadratic equations using, factorization, completing the square, quadratic formula refer to basic engineering mathematics by J.O Bird, Engineering mathematical by K.A STROUD etc

2.3.2.3. Self-Assessment

This section must be related with the Performance Criteria, Required Knowledge and Skills and the Range as stated in the Occupational Standards.

Q1. (a) Solve the following by factorization

(i) $x^2 + 8x + 7 = 0$

(ii) $x^2 - 2x + 1 = 0$

(b) Solve by completing the square the following quadratic equations

(i) $2x^2 + 3x - 6$

(ii) $3x^2 - x - 6 = 0$

Q.2 Simplify as far as possible

(i) $log(x^2 + 4x - 3) - log(x + 1)$

(ii) $2log(x - 1) - log(x^2 - 1)$

Q.3 Solve the following simultaneous equations by the method of substitution

$$x + 3y - z = 2$$
$$2x - 2y + 2z = 2$$
$$4x - 3y + 5z = 5$$

Q.4 Simplify the following

$$F = (2^{\frac{1}{2}}_{x} y^{\frac{1}{4}})^4 \div \sqrt{\frac{1}{9} x^2 y^6} \; x \; (4\sqrt{x^2 y^4})^{-1/2}$$

Describe quadratic equations.

Differentiate between the two methods of solving simultaneous equations

(i) By substitution

(ii) By elimination

Indices is the study of mathematical symbols and the rules for manipulating these symbols. True or false?

Quadratic equation is one in which the highest power of the unknown quantity is 2. True or false

2.3.2.4. Tools, Equipment, Supplies and Materials for the specific learning outcome

- Calculator
- Writing materials

2.3.2.5. References

- Basic Engineering Mathematics by J.O Bird
- Engineering Mathematics by K.A STROUD
- Technical mathematics book 2 by J.O bird

2.3.3. Learning Outcome No. 2. Apply trigonometry and hyperbolic functions

2.3.3.1. Learning Activities

Learning Outcome No. 2. Apply Trigonometry And Hyperbolic Functions	
Learning Activities	Special Instructions
• Perform calculations using trigonometric rules • Perform calculations using hyperbolic functions	

2.3.3.2. Information Sheet No 2/LO2.

Trigonometry is the branch of mathematics which deals with the measurement of sides and angles of triangles and their relationship with each other. Two common units used for measuring angles are degrees and radians.

Trigonometric ratios

The three trigonometric ratios derived from a right - angled triangle are the same, cosine and tangent refer to basic engineering mathematics by J.0 Bird to read move about trigonometry ratios.

Solution of right-angled triangles

To solve a triangle means to find the unknown sides and angles, this is achieved by using the theorem of Pythagoras and or using trigonometric ratios.

Example

In a triangle PQR shown below find the length of PQ and PR

$$t\ on\ 38^0 = \frac{PQ}{QR} = \frac{PQ}{7.5}$$

Hence

$$\begin{aligned} PQ &= 7.5\ X\ \tan 38^0 \\ &= 7.5\ x\ 0.7813 \\ &= 5.86\ cm \end{aligned}$$

$$\cos 38^0 = \frac{QR}{PR} = \frac{7.5}{PR}$$

$PR = \frac{7.5}{cos 38^0} = \frac{7.5}{0.7880}$ =9.518cm

For more worked examples refer to basic engineering mathematics by J.) Bird. Also use it to learn about angles of elevation and depression.

Compound angle formulae

Sin $(A \pm B) = Sin\ A \cos B \pm Cos\ A \sin B$

$Cos(A \pm B) = Cos\ Acos\ B \mp Sin\ A\ Sin\ B$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \, tanB}$$

Refer to technician mathematics book 3 by J.O Bird and learn more about compound angle formulae

Conversion of a coswt + bsinwt into the general form $R\ Sin(wt + 2)$

Let a Coswt + bSinwt = $R\ Sin(wt{+}\propto)$

Expanding the right hand side using the compound angle formulae gives

Acoswt + bSinwt = $R(Sinwt\ Cos \propto\ + coswt\ sin \propto)$

$= R \cos \propto\ |Sinwt + R\ Sin\ \propto Coswt$

Equating the coefficients of Coswt on both sides gives.

$a = R\ Sin \propto i.e\ Sin\ \propto = \frac{a}{R}$

Equating the coefficients of Sinwt on both sides gives

$b = R\ cos \propto\ \ i.e\ Cos\ \propto = \frac{b}{R}$

If the values of $a\ and\ b$ are known, the values of $R\ and\ \alpha$ can be calculated.

From Pythagoras theorem.

Diagram

$R = \sqrt{a^2} + b^2$

And from trigonometric ratios,

$\propto = tan^{-1}(\frac{a}{b})$

Example

Express 3 $Sin\ \theta + 4\ Cos\theta$ in the general form $R\ Sin(\theta + \alpha)$

Let 3 $Sin\ \theta + 4\ Cos\ \theta = R\ Sin\ (\theta + \alpha)$

Expanding the right hand side using the compound angle formulae gives

$3\ Sin\ \theta + 4\ Cos\ \theta = R\ [Sin\theta\ Cos\alpha + Cos\ \theta\ Sin\alpha\ ]$

$= R\ cos\alpha\ Sin\ \theta + R\ Sin\ \alpha\ Cos\ \theta$

Equating the coefficient of:

$Cos\theta; \;== R\, Sin\, \alpha\; i.e\; Sin\, \alpha = \dfrac{4}{R}$

Sin θ : $\quad 3 \;=\; R\, Cos\, \alpha \;\; i.e \;\; Cos\, \alpha = \frac{3}{R}$

This the values of $R\; and\; \alpha$ can be evaluated.

Diagram

$$R = \sqrt{4^2 + 3^2 = 5}$$

$$\alpha = tan^{-1}\frac{4}{3} = 53.13^0 or\;\; 233.13^0$$

Since both $Sin\, \alpha\; and\; Cos\, \alpha$ are positive, r lies in the first quadrant where all are positive, Hence 233.13^0 is neglected.

Hence

$3\, Sin\theta + 4\, Cos\, \theta = 5\, Sin\, (\theta + 53.13^0)$

Example

Solve the equation $3\, Sin\, \theta + 4\, Cos\, \theta = 2$ for values of θ between 0^0 and 360^0 inclusive

Soln:

From the example above

$3\, Sin\theta + 4\, Cos\, \theta = 5\, Sin\, (\theta + 53.13^0)$

Thus

$5\, Sin(\alpha + 53.13^0) = 2$

$Sin(\theta + 53.13^0) = \dfrac{2}{5}$

$\theta +\; 53.13^0 = Sin^{-1}\, 2/5$

$\theta +\; 53.13^0 =\; 23.58^0$ or 156.42^0

$\theta =\; 23.58^0 -\; 53.13^0 = -29.55^0$

$=330.45^0$

OR $\theta = 156.42^0 - 53.13^0$

$=103.29^0$

Therefore the roots of the above equation are 103.29^0 *or* 330.45^0

For more worked examples refer to Technician mathematics book 3 by J.) Bird.

Double /multiple angles
For double and multiple angles refer to Technician mathematics by J.O Bird

Factor Formulae
For worked exampled refer to Technic mathematics book 3 by J. O Bird, Pure mathematics by backhouse and Engineering mathematics by KA Stroud.

Half –angle formulae
Refer to pure mathematics by backhouse and Engineering mathematics by K.A STROUD

Hyperbolic functions
Definition of hyperbolic functions, $Sinhx\ cosh\ x\ and\ tanh\ x$

- Evaluation of hyperbolic functions
- Hyperbolic identifies
- Osborne's Rule
- Solve hyperbolic equations of the form a $coshx + bSinhx = C$

For all the above refer, to Engineering mathematics by K.A STROUD

2.3.3.3. Self-Assessment
Q1. A surveyor measures the angle of elevation of the top of a perpendicular building as19^0. He moves 120m nearer the building and measures the angle of elevation as47^0. Calculate the height of the building to the nearest meter.

Q2. Solve the equation $5\cos\theta + 4\,Sin\,\theta = 3$ for values of θ between 0^0 and 360^0
Inclusive.

Q3. Prove there identifies
$(i)\ Cash\ 2x = cash^2x + Sinh^2x$
$(ii) Sinh(x+y) Sinh\ Coshy + coshy\ Sinhx$
Q.4 Solve the equation
$3\ Sin\ hx + 4\ Coshx = 5$
Define trigonometry

5. Describe trigonometric ratios
6. Differentiate various hyperbolic functions
7. Perform calculations on various hyperbolic functions
8. Trigonometry is the branch of mathematics which deals with the measurement of sides and angles of triangles and their relationship with each other. True or false?
9. Hyperbolic function is in the form sinX, TanX. True or False?

2.3.3.4. Tools, Equipment, Supplies and Materials for the specific learning outcome

Writing materials

Geometrical set

Calculator

2.3.3.5. References

- Basic engineering mathematics by J.O Bird
- Technical mathematics book 3 by J.O Bird
- Pure mathematics book 1 by Backhouse
- Engineering mathematics by K a Stroud

2.3.4. Learning Outcome No.3 Apply Complex Number

2.3.4.1. Learning Activities

Learning Outcome No. 3. Apply Complex Number	
Learning Activities	Special Instructions
• Preparing complex numbers using ARgand diagrams • Performing operations involving complex number • Performing calculations involving complex number using De Moirés theorem	

2.3.4.2. Information Sheet chapter 2/LO3

Apply Complex Number

A number of the form $a + ib$ is called complex number where $a\ and\ b$ are real numbers and $i = \sqrt{-1}$ we call $'a'$ the real part and 'b' the imaginary part of the complex $a + ib$ if $a = o$ then ib is said to be purely imaginary, if $b = 0$ the number is real.

Pair of complex number $a + ib$ are said to be conjugate of each other.

Addition and subtraction of complex numbers

Addition and subtraction of complex numbers is achieved by adding or subtracting the real parts and the imaginary parts.

Example 1

$(4 + j5) + (3 - j2)$

$(4 + j5) + (3 - j2)$=$4 + j5 + 3 - j2$

=$(4 + 3) + j(5 - 2)$

=$7 + j3$

Example 2

$$(4 + j7) - (2 - js) = 4 + j7 - 2 + js = (4 - 2) + j(7 + 5)$$
$$= 2\ + j12$$

Multiplication of complex numbers

Example 1

$(3 + j4)(2 + j5)$

$6 + j8 + j15 + j^2 20$

$6 + j23 - 20\ (since j^2 = -1)$

=$-14 + j^{23}$

Examples 2

$(5 + j8)(5 - j8)$

$(5 + j8)(5 - j8) = 25 + j40 - j40 - j^2 64$

$= 25 + 64$

=89

A pair of complex numbers are called conjugate complex numbers and the product of two conjugate. Complex numbers is always entirely real. $cos\theta + jsin\theta$

Argand diagram

Although we cannot evaluate a complex number as a real number, we can represent diagrammatically in an argand diagram. Refer to Engineering Mathematics by K.A Stroud to learn more on how to represent complex numbers on an argand diagram. Use the same back learn three forms of expressing a complex number.

Demoivre's Theorem

Demoivre's theorem states that $[r(cos\theta + jsin\theta)]^n = r^n(cosn\theta + jsinn\theta)$

It is used in finding powers and roots of complex numbers in polar

Example

Find the three cube roots of $z = 5(cos225^0 + jsin225^0)$

$$Z_1 = Z^{\frac{1}{3}}\left(Cos\frac{225^0}{3} + jsin\frac{225^0}{3}\right)$$

$1.71\ (cos75^0 + jsin75^0$

$z_1 = 1.71\ (cos75^0 + jSin75^0)$

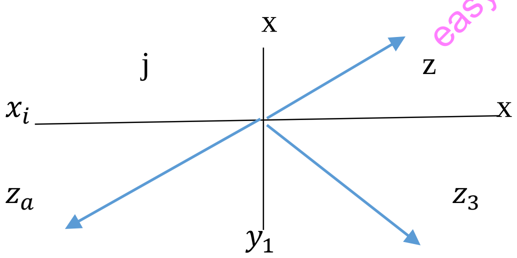
Cube roots are the same size (modules) i.e 1.71 and separated at intervals of $\frac{360^0}{3}$, i.e 120^0

$z_{1=1.71\,/}\ 75^0$

$z_{2=1.71}\ \cos(195^0 + jSin195^0$

$z_s = 1.71\ (315^+ + jSin315^0)$

Sketched Argand diagram.



Refer to engineering mathematics by K.A Stroud and learn more on how to find the expansion of

$cos^n\theta$ and $cos^n\theta$

LOCI problems

We sometimes required to find the locus of a point which moves in the Argand diagram according to some stated condition.

Examples

If $Z = x + jy$, find the equation of the locus $\left[\frac{z+1}{z-1}\right] = 2$

$sin\theta Z = x + jy.$

$Z + 1 = X + jy + 1 = (x + 1) + jy = r_1 angle\ \theta_1 = z_1$

$z - 1 = x + jy - 1 = (x - 1) + jy = r_2 r_1 angle\ \theta_2 = z_2$

$\therefore \frac{z+1}{z-1} = \frac{r_{1\theta_1}}{r_{2\theta_2}} = \frac{r_1}{r_2}\ \theta_1$ -θ_2

$\therefore \left(\frac{Z+1}{z-1}\right) = \frac{r_1}{r_2} = \left(\frac{z_1}{z_2}\right) = \frac{[(x+1)^2 + y^2]}{[(x-1)^2) + y^2]}$

$$\frac{[(x+1)^2 + y^2]}{(x-1)^2 + y^2}$$

$$\therefore \frac{(x+1)^2 + y^2}{(x-1)^2 + y^2} + 4$$

$\therefore (x+1)^2 + y^2 = 4\ ((x-1)^2 + y^2)$

$x^2 + 2x + 1 + y^2 = 4(x^2 - 2x + 1 + y^2)$

$=4x^2 - 8x + 4 + 4y^2$

$\therefore 3x^2$-10x+3+$3y^2 = 0$

2.3.4.3. Self-Assessment

1. Find the fifth roots of $-3 + j3$ in polar form and in exponential form
2. Determine the three cube roots of $\frac{2-j}{2+j}$ giving the results in a modulus/ argument form.

   Express the principal root in the form $a + jb$
3. If $z = x + jy,\ where\ x\ and\ y$ are real, show that the locus $\left(\frac{Z-2}{z+2}\right) = 2$ is a circle and determine its center and radius.
4. Describe De Moirés theorem
5. Perform calculations involving complex number using De Moirés theorem
6. Explain Argand diagram
7. A number of the form $a + ib$ is called complex number where $a\ and\ b$ are real numbers and $i = \sqrt{-1}$ TRUE OR FALSE?
8. Demoivre's theorem states that $[r(cos\theta + jsin\theta)]^n = r^n(cosn\theta + jsinn\theta)$ TRUE OR FALSE?

2.3.4.4. Tools, Equipment, Supplies and Materials for the specific learning outcome

- Calculator
- Writing materials
- Geometrical set.

2.3.4.5. References

1. Technician Mathematics Book (4 and 5) by J.O Bird
2. Engineering Mathematics by K.A Stround
3. Mathematics for Engineers by Dass

2.3.5. Learning Outcome No. 4. Apply Co-ordinate Geometry
2.3.5.1. Learning Activities

Learning Outcome No. 4. Apply Co-Ordinate Geometry	
Learning Activities	Special Instructions
• Calculating polar equations using coordinate geometry • Drawing graphs of given polar equations using the Cartesian plane • Determining normal and tangents using coordinate geometry	

2.3.5.2 Information Sheet No. 2/LO4

Introduction

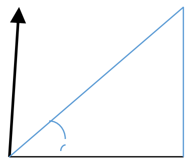
The position of a point in a plane can be represented in two forms

i) Cartesian co-ordinate (x, y)

ii) Polar co-ordinate (r, θ)

The position of a point in the corresponding axis can therefore generate Cartesian and polar equations which can easily change into required form to fit the required result.

Example



Convert $r^2 = sin\theta$ into Cartesian form.

$$cos\theta = \frac{x}{y}$$

$$sin\theta = \frac{y}{x}$$

Form Pythagoras theorem $r^2 = x^2 + y^2$

$$r^2 = sin\theta$$

$$(x^2 + y^2) = \frac{y}{x}$$

$$(x^2 + y^2)r = y$$

$$(x^2 + y^2)(x^2 + y^2)^{\frac{1}{2}} = y$$

$$(x^2 + y^2)^{\frac{3}{2}} = y$$

Example 2

Find the Cartesian equation of

(i) $r = a(1 + 2cos)$ (ii) $rcos(\theta - \alpha) = p$

[The $Cos\theta$ suggest the relation$X = COS\theta$, so multiplying through by r}

$$\therefore r^2 = a(r + 2rcos\theta)$$

$$\therefore x^2 + y^2 = a\left(\sqrt{(x^2 + y^2)} + 2x\right)$$

$$\therefore x^2 + y^2 + 2x = a\sqrt{(x^2 + y^2)}$$

Therefore the Cartesian equation of $r = a(1 + 2cos)$ is $(x^2 + y^2 - 2ax)^2 = a^2(x^2 + y^2)$

(ii) $rcos(\theta - \alpha) = p$

$cos(\theta - \alpha)$ May be expanded

$\therefore rcos\theta cos\alpha + rsin\theta sin\alpha = p$

(iii) Therefore the Cartesian equation of $rcos(\theta - \alpha) = p$ is $xcos\alpha + ysin\alpha = p$

Example 3

Find the polar equation of the circle whose Cartesian equation is $x^2 + y^2 = 4x$

$x^2 + y^2 = 4x$

Put $x = rcos\theta, y = rsin\theta$, then

$r^2cos^2\theta + r^2sin^2\theta = 4rcos\theta$

$\therefore r^2 = 4rcos\theta$

Therefore the polar equation of the circle is $r^2 = 4rcos\theta$.

For more information on the conversion of Cartesian equation to polar equation and vice versa refer to pure mathematics by J.K Backhouse

2.3.5.3. Self-Assessment

1) Obtain the polar equation of the following loci
   (i) $x^2 + y^2 = a^2$
   (ii) $x^2 - y^2 = a^2$
   (iii) $y = 0$
   (iv) $y^2 = 4a(a - x)$
   (v) $x^2 + y^2 - 2y = 0$
   (vi) $xy = c^2$
2) Obtain the Cartesian equation of the following loci
   (i) $r = 2$
   (ii) a(1+cosθ)
   (iii) $r = acos\theta$
   (iv) $r = atan\theta$
   (v) $r = 2a(1 + sin2\theta)$
   (vi) $2r^2sin2\theta = c^2$
   (vii) $\frac{l}{r} = 1 + 8cos\theta$
   (viii) $r = 4acot\theta cosec\theta$
3) Differentiate between Cartesian co-ordinate and Polar co-ordinate
4) Perform calculations on polar equations
5) The position of a point in a plane can be represented in two forms: Cartesian co-ordinate $(x, y) or$ Polar co-ordinate (r, θ) TRUE OR FALSE?
6) The equation $r^2 = x^2 + y^2$ illustrates Pythagoras theorem. TRUE OR FALSE?

2.3.6 Learning Outcome No.5 Carry out Binomial Expansion
2.3.6.1 Learning Activities

Learning Outcome No. 5. Carry Out Binomial Expansion	
Learning Activities	Special Instructions
• Determining roots of numbers using binomial theorem • Determining errors and small changes using bionomical theorem • To cover the Performance Criteria statements • Trainees to demonstrate knowledge in relation to; The *Range* in the Occupational Standards and *Content* in the curriculum	

Carry out Binomial expansion

Binomial is a formula for raising a binomial expansion to any power without lengthy multiplication. It states that the general expansion of $(a+b)^n$ is given as

$$(a+b)^n = a^n b^0 + na^{n-1}n^1 + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \ldots$$

Where n can be a fraction, a decimal fraction, positive or negative integer.

Example 1

Use binomial theorem to expand $(2+x)^3$

Solution

$$(a+b)^n = a^n b^0 + na^{n-1}n^1 + \frac{n(n-1)a^{n-2}b^2}{2!} + \frac{n(n-1)(n-2)a^{n-3}b^3}{3!} + \ldots$$

$$A = 2, b = x \text{ and } n = 3$$

$$(2+x)^3 = 2^3x^0 + 32^2x^1 + \frac{3(3-1)2^1x^2}{2!} + \frac{3(3-1)(3-2)2^0x^3}{3!} + \ldots$$

$$= 18 + 12x + 6x^2 + x^3$$

For more examples on positive power refer to Technician Mathematic Book by J.O Bird.

Binomial theorem for any index

It has been shown that

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \ldots.$$

The series may be continued indefinitely for any value of n provide $-1 < x < 1$

Example

Use the binomial theorem to expand $\frac{1}{1-x}$ in ascending power of x as far as the term in x^3.

Solution

Since $\frac{1}{1-x}$ may be written$(a - x + x)^{-1}$, the binomial theorem may be used. Thus

$$(1-x)^{-1} = 1 + -1(-x) + \frac{-1(-2)}{2!}x^2 + \frac{-1(-2)(-3)}{3!} + \ldots.$$

$$\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$$

Provided $-1 < x < 1$

Practical application of binomial theorem

Example1

The radius of a cylinder is reduced by 4% and its height increased by 2%. Determine the appropriate percentage change in its volume neglecting products of small quantities.

Solution

Volume $V = \pi r^2 h$

Let original values be, $radius = r$

$$Height = h$$

New values $radius = (1 - 0.04)r$

$$Height = (1 + 0.02)h$$

New volume $= \pi(1 - 0.004)^2 r^2 (1 + 0.04)h)$

Using binomial theorem, $(1 - 0.04)^2 = 1 - 2(0.04) + (0.04)^2 = 1 - 0.08$

$=\pi r^2 h(1 - 0.08)(1.02) = \pi r^2 h(0.94)$

Percentage change $= \frac{(0.94-1)100\%}{1} = -6\%$

The new volume decreased by 6%

2.3.6.3. Assessment

1. Expand as far as the third term and state the limits at which the expansions are valid.
   (i) $\frac{1}{(1+2x)^3}$
   (ii) $\sqrt{4+x}$
2. Show that if higher power of x are neglected

$$\sqrt{\frac{1+x}{1-x}} = 1 + x + \frac{x^2}{2}$$

3. The second moment of area of a rectangular section through its centroid is given by$\frac{bl3}{12}$. Determine the appropriate change in the second moment of area if b is increased by 3.5% and l is reduced by 2.5%
4. Explain binomial theorem
5. Describe binomial expansion
6. Identify roots of numbers using binomial theorem
7. Bionomical theorem can be used to identify small changes and errors TRUE OR FLASE?
8. Binomial is a formula for raising a binomial expansion to any power without lengthy multiplication. TRUE OR FALSE?

2.3.6.5. References

1. John Bird- Higher Engineering Mathematics sixth edition
2. Pure Mathematics: JK BackHouse

2.3.7. Learning Outcome No. 6. Apply Calculus

2.3.7.1. Learning Activities

Learning Outcome No. 6. Apply Calculus	
Learning Activities	Special Instructions
• Determining Derivatives from first principles • Determining derivatives of inverse trigonometric functions • Determining the rate of change and small changes using differentiation • Determining integrals of algebraic functions • Determining integrals of trigonometric functions • Determining integrals of logarithmic functions • Determining integrals of hyperbaric and inverse hyperbaric functions.	

2.3.7.2 Information Sheet No. 2/ LO6

Calculus is a branch of mathematics involving calculations dealing with continuously varying functions. The subject falls into two parts namely differential calculus (differentiation) and integral calculus (integration)

Differentiation

The central problem of the differential calculus is the investigation of the rate of change of a function with respect to charges in the variables on which it depends.

Differentiation from first principles.

To differentiate from first principles means to find $f'(x)$ using the expression.

$$f'(r) = \lim_{\delta x \to 0} \left\{ \frac{f(x+\delta x)}{\delta x} \right\}$$

$$\delta x \to \ 0 \left\{ + \frac{(x+\delta x) - f(x)}{\delta x} \right\}$$

$$f(x) = x^2$$

$$f(x+\delta x) = (x+\delta x)^2 = x^2 + 2x\delta x + \delta x^2$$

$$f(x+\delta x) - f(x) = x^2 + 2\ x\delta x + \delta x^2 - x^2$$

$=2x\delta x + \delta x^2$

$$\frac{f(x+\delta x) - f(x)}{\delta x} = \frac{2x\delta x + \delta x^2}{\delta x}$$

$=2x + \delta x$

As $\delta x \to 0$, $\frac{f(x+\delta x)-f(x)}{\delta x} \to 2x + 0$

$\therefore f'(x) = \lim_{\delta x \to 0} \left\{ \frac{f(x+\delta x)-f(x)}{\delta x} \right\} = 2x$

At $x = 3$, the gradient of the curve i.e $f'(x) = 2(3) = 6$
Hence if $f(x) = x^2$, $f'(x) = 2x$. The gradient at $x = 3$ is 6

Methods of differentiation

There are several methods used to differentiate different functions which include:

(i) Product Rule
(ii) Quotient Rule
(iii) Chain Rule
(iv) Implicit Rule

Example:

Determine ${dy}/{dx}$ given that $y = x^2 Sinx$

Solution

From product rule: $uv(x) = u\frac{dv}{dx} + v\frac{du}{dx}$

$u = x^2$ and $v = Sinx$

$\frac{du}{dx} = 2x$ $\qquad$ $\frac{dv}{dx} = Cosx$

$\therefore \frac{dy}{dx} = x^2(Cosx) + Sin\,(2x)$

$= x^2 Cos\,x + 2x\,Sin\,x$

$$y = \frac{x^2 + 1}{x - 3}$$

Soln. Using Quotient rule:

$$\frac{u(x)}{v(x)} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

$u = x^2 + 1$ $\qquad$ $v = x - 3$

$\frac{du}{dx} = 2x$ $\qquad$ $\frac{dv}{dx} = 1$

$\therefore dy/dx = \frac{(x-3)(2x)-(x^2+1)(1)}{(x-3)^2}$

$= \frac{2x^2-6x-x^2-1}{(x-3)^2}$

$= \frac{x^2-6x-1}{(x-3)^2}$

$$\frac{x^2 - 6x - 1}{x^2 - 6x + 9}$$

For more examples on the cases of application of the other, highlighted rates refer to Engineering Mathematics by K Stroud.

Application of differentiation

Differentiation can be used to determine velocity and acceleration of a moving body. It can also be applied to determine maximum and minimum values.

Example: A rectangular area in formed using a pieced of wire 30cm long. Find the length and breadth of the rectangle if it is to enclose the maximum possible area.

Solution.
Let the dimension a rectangle be $x \; and \; y$
Perimeter of rectangle = $2x + 2y = 36$
i.e $x + y = 18$………………(i)
Since it's the maximum area that is required a formula for the area A must be obtained in terms of one variable only.
Area = $A = xy$
From equation (i) $y = 18 - x$
Hence $A = x(18 - x) = 18x - x^2$
Now that an expression for the area has been obtained in terms of one variable it can be differentiated with respect to that variable
$\frac{dA}{dx} = 18 - 2x \; for \; maximum$ or minimum value i.e $x = 9$

$$\frac{d^2A}{dx^2} = -2, which \; is \; negative \; giving \; a \; maximum \; value$$

$$y = 18 - x = 18 - 9 = 9$$

Hence the length and breadth of the rectangle of maximum area both 9 cm i.e a square gives the maximum possible area for a given perimeter length When perimeter is 36cm maximum area, possible is $81cm^2$.

Integration
Process of integration reverses the process of differentiation. In differentiation if $f(x) = x^2$ then $f'(x) = 2x$.
Since integration reverse the prices of moving from $f(x)$ to f'(x), it follows that the integral of $2x \; is \; x^2 \; i.e \; its \; the \; process \; of \; moving \; from \; f'$(x) to $f(x)$. Similarly if $y = x^2$ then $\frac{dy}{dx} = 3x^2$. Reversing this process shows that the integral of $3x^2$ and x^3.
Integration is also the process of summation or adding parts together and on elongate $'s'$ shown a $\int$ is used to replace the words 'integrated of "Thus $\int 2x$=x^2 and $\int 3x^2 = x^3$
Refers to engineering mathematics by K.A strond and learn those on definite and indefinite integrals.

Methods of integration and application of integration
The methods available are:

a. By using an algebraic substitution
b. Using trigonometric identifies and substitutions

c. Using partial fraction
d. Using the $t = tan\frac{\theta}{2}$ substitution
e. Using integration by parts

Refer to Engineering mathematics by K.A strond and learn more about methods of integration,
Also we the above stated book to learn more on application on integration to find areas, volumes of revolutions etc

2.3.7.3. Self-Assessment

1. Find the co-ordinator, of the points on the curve
$$y = \frac{1/3(5-6x)}{3x^2+2}$$
Where the gradient is zero.

2. If $y = \frac{4}{3x^3} - \frac{2}{x^2} + \frac{1}{3x} - \sqrt{x}$. Find $\frac{d^2y}{dx}$ $and \frac{d^3y}{dx}$

3. Find $\int cos6x\ Sin\ 2x\ \ dx$

4. Evaluator $\int_3^4 \frac{x^3-x^2-5x}{x^2-2x+2}dx$
5. Explain calculus
6. Perform calculations on inverse trigonometric functions
7. Differentiate algebraic functions, trigonometric functions and logarithmic functions
8. Perform calculations on integrals of hyperbaric and inverse hyperbaric functions.
9. Cross product is a method of differentiation. TRUE OR FASE?
10. Use of partial fraction is a method of differentiation. TRUE OR FLASE

2.3.7.4. Tools, Equipment, Supplies and Materials for the specific learning outcome

- Writing materials
- Calculator

2.3.7.5. References

1. Technician Mathematics Book (4 and 5) by J.O Bird
2. Engineering Mathematics by K.A Stround
3. Mathematics for Engineers by Dass

2.3.8. Learning Outcome No. 7. Solve Ordinary Differential Equations

2.3.8.1. Learning Activities

Learning Outcome No. 7. Solve Ordinary Differential Equations	
Learning Activities	Special Instructions
• Solve first order and second order differential equations using the method of undetermined coefficients. • Apply boundary conditions to find the particular solution	• Find general solution • Find particular solution

2.3.8.2. Information Sheet No. 2/ LO7

An equation involves differential co-efficient is called a differential equation examples.

(i) $\frac{dy}{dx} = \frac{1+x^2}{1-y^2}$

(ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 0$

The order of a differential equation is the order of the highest differential is coefficient present in the equation.

Differential equations represent dynamic relationship i.e quantities that change, and are thus

An equation which involve differential co-efficient is called a differential equation.

Example:

(i) $\frac{dy}{dx} = \frac{1+x^2}{1-y^2}$

(ii) $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 0$

The order of a differential equation is the order of the highest differential coefficient present in the equation.

A differential equation represent dynamic relationships, ie quantities that change, and are thus frequently occurring in scientific and engineering problems.

Formation of a differential equation

Differential equations may be formed in practice from a consideration of the physical problem to which they refer. Mathematically, they can occur when arbitrary constants are eliminated from a given function.

Example

Consider $y = Asinx + Bcosx$, where A and B are two arbitrary constants. If we differentiate, we get

$\frac{dy}{dx} = ACosx - BSinx$ and $\frac{d^2y}{dx^2} = -ASinx - BCosx = -(aSinx + BCosx)$

i.e $\frac{d^2y}{dx^2} = -y$

∴ $$\frac{d^2y}{dx^2} - y = 0$$

This is a differential equation of the second order.

For the formation of first order differential equations, refer to Technician Mathematics 4 and 5 by J.O Bird.

Types of first order differential equations:

1. By separating the variables
2. Homogeneous first order differential equations
3. Linear differential equations
4. Exact differential equations

Refer to Engineering Mathematics by K.A Stround, Technician 4 and 5 by J.O Bird for worked out examples and further exercises.

Application of first order differential equations.

Differential equations of the first order have many applications in engineering and science.

Example:

The rate at which a body cools is given by the equations $\frac{d\theta}{dt} = -k\theta$ where θ the temperature of the body above the surroundings is and k is a constant. Solve the equation for θ given that $t = 0$,

$\theta = \theta_0$

Solution

$$\frac{d\theta}{dt} = -k\theta$$

Rearr
angin
g
gives
$dt = \frac{-1}{k\theta}$

Integrating both sides gives $\int dt = \frac{-1}{k}\int \frac{d\theta}{\theta}$

i.e $t = \frac{-1}{k} ln\theta + c$…………………..(i)

Substituting the boundary conditions $t = 0$, $\theta = \theta_0$ to find c gives

$$0 = \frac{-1}{k} ln\theta_0 + c$$

i.e $$c = \frac{1}{k} ln\theta_0$$

Substituting $c = \frac{-1}{k} ln\theta_0$ in equation (i) gives

$$t = \frac{-1}{k} ln\theta + \frac{1}{k} ln\theta_0$$

$$t = \frac{1}{k}(ln\theta_0 + ln\theta) = \frac{1}{k}\ln\left(\frac{\theta_0}{\theta}\right)$$

$$kt = ln\left(\frac{\theta_0}{\theta}\right)$$

$$e^{kt} = \frac{\theta_0}{\theta}$$

$$e^{-kt} = \frac{\theta}{\theta_0}$$

Hence $$\theta = \theta_0 e^{-kt}$$

Further problems on application of differential equations may be found in Engineering Mathematics by K.A Stround, Technician 4 and 5 by J.O Bird.

Formation of the second order differential equation

For, formation of second order differential equations refer to *Engineering Mathematics by K.A Stround, Technician 4 and 5 by J.O Bird.*

Application of second order differential equations

Many applications in engineering give rise to the second order differential equations of the form

$$a\frac{d^2y}{dx^2} + b\frac{dy}{dx} + cy = f(x)$$

Where a, b, c are constants coefficient and $f(x)$ is a given function of x.

Example include:

(1) Bending of beams
(2) Vertical oscillations and displacements
(3) Damped forced vibrations

For more worked examples refer to Engineering Mathematics by K.A Stround, Technician 4 and 5 by J.O Bird.

2.3.8.3 Self-Assessment

Q1. Solve the following equations:

(i) $x(y-3)\frac{dy}{dx}4y$

(ii) $(xy + y^2) + (x^2 - xy)\frac{dy}{dx} = 0$

(iii) $\frac{dy}{dx} + ytanx = sinx$

Q2. The change, q, on a capacitor in an LCR circuit satisfies the second order differential equation

$$L\frac{d^2q}{dt^2} + b\frac{dq}{dt} + \frac{1}{c}q = E$$

Show that if $2L = cR^2$ the general solution of this equation is

$$q = e^{\frac{-t}{cR}}\left(Acos\frac{1}{cR}t + Bsin\frac{1}{cR}t\right) + cE$$

If $i = \frac{dq}{dt} = 0$ and $q = 0$ when $t = 0$. Show that the current in the circuit is

$$i = \frac{2E}{R}e^{\frac{-t}{cR}}sin\frac{1}{cR}$$

3. Explain differential equations
4. Describe method of undetermined coefficients
5. Perform calculations on first order and second order differential equations using the method of undetermined coefficients
6. An equation which involve differential co-efficient is called a differential equation. TRUE OR FALSE?
7. Linear differential equations is not a first order differential equation. TRUE OR FALSE?

2.3.8.4. Tools, Equipment, Supplies and Materials for the specific learning outcome

- Calculator
- Writing materials

2.3.8.5. References

1. Technician Mathematics Book (4 and 5) by J.O Bird
2. Engineering Mathematics by K.A Stroud
3. Mathematics for Engineers by Dass

2.3.9 Learning Outcome No. 8. Apply Laplace transforms

2.3.9.1 Learning Activities

Learning outcome No. 8. Apply Laplace transforms	
Learning Activities	Special Instructions
• Derive Laplace transform from first principle. • Solve Laplace transforms using initial and final value theorem. • Determine inverse using Laplace transform using partial fractions, • Solve differential equations by using Laplace transform • To cover the Performance Criteria statements • Trainees to demonstrate knowledge in relation to; The *Range* in the Occupational Standards and *Content* in the curriculum	• Rule of partial fractions

2.3.9.2. Information Sheet No 2/LO8

Definition

The Laplace transform of a function $F(t)$ is denoted by $\mathcal{L}[F(t)]$ and is defined as the integral of $F(t)e^{-st}$ between the limits $t = 0$ and $t = \infty$.

i.e $\mathcal{L}[F(t)] = \int_0^\infty F(t)e^{-st}$ dt

in determining the transform of any function, you will appreciate that the limits are substituted for t, so that the result will be a function of s.

$$\therefore\ \mathcal{L}[F(t)] = \int_0^\infty F(t)e^{-st}\,\mathrm{dt} = f(s)$$

Deriving the Laplace transform from the first principles

To find the Laplace transform from first principles.

Example

To find the Laplace transform of $F(t) = a$ (constant)

$$\mathcal{L}(a) = \int_0^\infty a\, e^{-st}dt = \left[a\frac{e^{-st}}{-s}\right]_0^\infty = -\frac{a}{s}[e^{-st}]_0^\infty$$

$$= \frac{-a}{s}[0-1] = \frac{a}{s}$$

$$\therefore \mathcal{L}(a) = \frac{a}{s}$$

Example

To find the Laplace transform of $F(t) = e^{at}$

Solution

$$\mathcal{L}(e^{at}) = \int_0^\infty e^{at}\, e^{-st} dt$$

$$= \int_0^\infty e^{-(s-a)t}\, dt$$

$$= \left[\frac{e^{-(s-a)t}}{-(s-a)}\right]_0^\infty$$

$$= \frac{-1}{s-a}(0-1)$$

$$= \frac{1}{s-a}$$

$$\therefore \mathcal{L}(e^{at}) = \frac{1}{s-a}$$

For more worked examples on how to derive the Laplace transform from first principles refer to Advanced Engineering Mathematics by K.A Stround.

Inverse Transforms

Here we have the reverse process, ie given a Laplace transform, we have to find the function of t to which it belongs.

For example, we know that $\frac{a}{s^2+a^2}$ is the Laplace transform of $Sinat$ so we can write

$$\mathcal{L}^{-1}\left(\frac{a}{s^2+a^2}\right) = Sinat$$

The symbol $\mathcal{L}^{-1}$ indicating the inverse transform and not the reciprocal.

For worked examples on how to find inverse of the Laplace transform refer to Further Engineering Mathematics by K.A Stround.

Solution of differential equation by Laplace transforms.

To solve a differential equation by Laplace transform we go through four distinct stages.

(1) Re-write the equation in terms of Laplace transform
(2) Insert the given initial conditions
(3) Rearrange the equation algebraically to given the transform of the solution.

For worked examples and exercises refer to Further Engineering Mathematics by K.A Stround.

2.3.9.3. Self-assessment

Q1. Derive the Laplace transform of each of the following expressions

(i) $f(t) = Sin2t$

(ii) $f(t) = Cosat$

(iii) $f(t) = e^{at}$

Q2. Find the inverse Laplace transform of each of the following expressions

(i) $\frac{4}{s^2+4s+5}$

(ii) $\frac{s+5}{s^2+10s+29}$

(iii) $\frac{2s-3}{s^2+6s+10}$

Q3. Solve the following differential equations using Laplace transforms

(i) $x'' + x' + 2x = 4Cos2t$

(ii) $x'' - 4x = 4t$

4. Describe Laplace transform from first principle
5. Explain inverse transform
6. Differentiate initial and final value theorem.
7. Perform calculations using Laplace transform
8. The Laplace transform of a function $F(t)$ is denoted by $\mathcal{L}[F(t)]$ and is defined as the integral of $F(t)e^{-st}$ between the limits $t = 0$ and $t = \infty$. TRUE OR FALSE
9. Insert the given initial conditions is not one of the stages of solving a differential equation by Laplace transform. TRUE OR FALSE

2.3.9.4 Tools, equipment's

- Calculator
- Laplace transform Tables
- Writing material

2.3.9.5 References

1. Further Engineering Mathematics by K.A Stround
2. Mathematics for Engineers by Dass

2.3.10. Learning Outcome No. 9. Apply Power Series

2.3.10.1 Learning Activities

Learning Outcome No. 9. Apply Power Series	
Learning Activities	Special Instructions
• Obtaining power series using Taylor's the Obtaining power series using Taylor's theorem • Obtaining power series using Maclaurins Theorem.	

2.3.10.2 Information Sheet No. 2/ LO9

The power series of claurins theorem different functions can be carried out using two theorems.

(i) Taylor's Theorem

(ii) Maclarins theorem

Taylor's series states that;

$$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots$$

Examples

Express$Sin(x+h)$ as a series of powers of h and hence evaluates $Sin\ 44^0$ correct to four decimal places.

Soln

$f(x+h) = f(x) + hf'(x) + \frac{h^2}{2!}f''(x) + \frac{h^3}{3!}f'''(x) + \cdots$

$f(x) = Sinx$

$f'(x) \ = cos\ x$

$f''(x) = -sinx$

$f'''(x)$ =-cosx

$f^{iv}(x) = \sin x$

$\therefore Sin(x+h) = \sin x + hcos\ x - \frac{h^2}{2}$ sin x - $\frac{h^3}{6}\ cos\ x$ ....

$Sin44^0 = \sin(45^0 - 1^0)$

=$Sin(^{\pi}/_4) - 0.01745$

=$\sin{}^{\pi}/_4 - 0.01745\ cos\frac{\pi}{4}$ - $\frac{0.01745^2}{2}\ sin{}^{\pi}/_4 + \frac{0.0174s^3}{6}\ cos{}^{\pi}/_4$

$But\ sin45 = cos\,45 = 0.707$

$= 0.707\ (1 - 0.01745 - 0.0001523 + 0.0000009)$

$= 0.707(0.982395)$

$= 0.69466$

$0.6947(4dp)$

For the use Maclaurns theorem refer to Engineering Mathematics by Strond.

2.3.10.3. Self-Assessment

1. Use Manchurians theorem to expand $ln(3x + 1)$. Hence use the expansion to evaluate $\int_0^1 \frac{ln(3x+1)}{x^2} dx$ to four decimal places
2. Use Taylors series to expand $\cos\left(\frac{\pi}{3} + h\right)$ in terms of h as far as h^3. Hence evaluate $cos68^0$ correct to four decimal places
3. Describe power series of claurins theorem
4. Differentiate between Taylor's Theorem and Maclarins theorem
5. Perform calculations on power series using Taylor's theorem
6. Perform calculations on power series using Maclaurins Theorem
7. Taylor's Theorem can be used to obtain power series. TRUE OR FALSE?
8. Taylor's series states that;
9. $f(x + h) = f(x) + hf'(x) + \frac{h^2}{2!} f''(x) + \frac{h^3}{3!} f'''(x) + \cdots TRUE\ OF\ FALSE?$

2.3.10.4. Tools, Equipment, Supplies and Materials for the specific learning outcome

- Advanced Calculator

2.3.10.5. References

1. Strond, 6th Edition Advanced Engineering mathematics

2.3.11. Learning Outcome No. 10. Apply statistics

2.3.11.1. Learning Activities

Learning Activities No. 10. Apply Statistics	
Learning Activities	Special Instructions
• Perform identification collection and organization of data • Perform interpretations analysis and presentation of data in appropriate format • Evaluate mean median mode and standard deviation obtained from the given data • Performing calculations based on laws of probability • Performing calculations involving probability distributions and mathematical expectation sampling.	

2.3.11.2. Information Sheet No. 2/ LO10

Statistics is discipline which deals with collection, organization, presentation and analysis of data.

Data consist of set of record observations that carry information on a particular setting with the availability of data.

A statistical exercise normally consist of four stages.

1. Collection of data
2. Organization and presentation of data in convenient form
3. Analysis of data to make their meaning clear
4. Interpretation of 1b results and the conclusion

In statistic we consider quantities that are varied. This quantities are referred as variables.

Variables are denoted by letters

Examples

- Heights
- Ages
- Weighs
- Times

Types of data;

1. Quantitative data
2. Qualitative data

Presentation of data

The aim of presenting data is to communicate information.

The type of presentation chosen depend on the requirement and the interest of people receiving that particular information.

Frequently the first stage in presenting is preparing a table.

Tabulation of data

Frequency distribution

Given a set of draw data we usually arrange into frequency distribution where we collect like quantities and display them by writing down their frequencies.

For those on data presentation refer to engineering mathematics by K.A stroud.

Measures of central tendencies

This is single value which used to represent entire set of data. It is typical value to which most observation fall closest than any other value.

There are mainly measures of central tendencies.

i. Arithmetic mean
ii. Median
iii. Mode

Refer to Engineering mathematics for Engineers by H.K Dass

Measurers of Dispersion

They include;

1. Range
2. Standard deviation
3. Quartiles

Normal Distribution

This is a continuous distribution. It is derived as the limiting for of the Binomial distribution for large values of n and p and q are not very small.

The normal distribution is given by the equation

$fx) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2}$

Where μ is the mean and σ is the standard deviation, $\pi = 3.14159$ and $e = 2.71828$

$$P(x_1 < x < x_2) = \int_{x_1}^{x_2} \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} dx \dots\dots\dots\dots\dots\dots\dots\dots.(1)$$

On substitution $z = \frac{x-\mu}{\sigma}$ in (1) we get $f(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}z^2} \dots\dots\dots\dots\dots\dots\dots\dots..(2)$

Here $mean = 0,\ standard\ deviation\ = 1$

Equation (2) is known as standard form of normal distribution.

Normal curve

Shown graphically: the probabilities of heads in 1 losses are

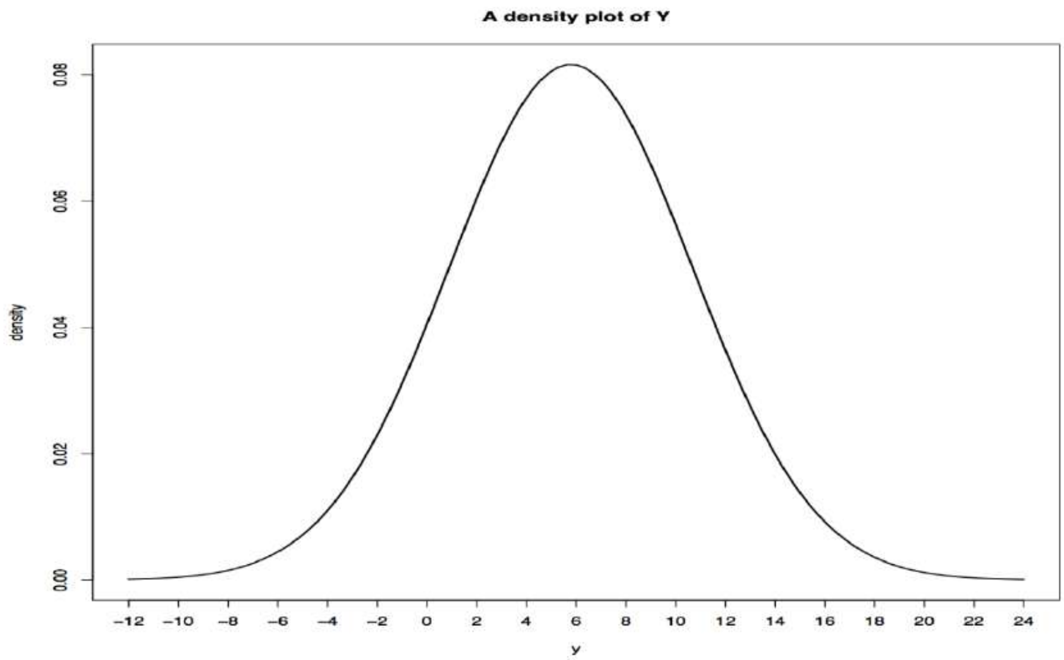


Figure 1: A density plot of Y

Area under the normal curve

By taking $z = \frac{x-\mu}{\sigma}$, the standard normal curve is formed.

The total area under this curve is 1. The area under the curve is divided into two equal parts by $z = 0$. Left had side area and right hand side area to $z = 0$ is 0.5. The area between the ordinate $z = 0$ and any other ordinate can be calculated.

Example 1

On the final examination in mathematics the mean was 72 and the standard deviation was15. Determine the standard scores of students receiving grades

a) 60 b) 93 c) 72

Solution

a) $z = \frac{x-\mu}{\sigma} = \frac{60-72}{15} = -0.8$

b) $z = \frac{93-72}{15} = 1.4$

c) $z = \frac{72-72}{15} = 0$

For more refer to Mathematics for Engineers by H.K Dass .

Read on Poison and standard deviation of Binomial distribution.

2.3.11.3 Self-Assessment

1. A machine produced components whose masses are normally distributed with mean μ and standard deviation σ if 89.8 % of the components have a mass of at least 88g and 3% have a mass less than 84.5g. Find the mean and the standard deviation of the distribution (6mks).
2. The diameters of bolts produced by a certain machine are distributed by a probability density function

$$f(x) = \begin{cases} kx(3-x), & 0 \le x \le 3 \\ 0 & Otherwise \end{cases}$$

Find the;

a. Constant$k$.

b. Probability that the diameter of a bolt-selected at a random will fall in the interval

$$1 < x < 2.5$$

c. Mean and the variance of the distribution (14mks)

3. Tumaini Ltd, is supplied with petrol once a week. The weekly demand, x hundreds of liters, has the probability density function

$$f(x) = \begin{cases} k(1-x), & 0 \le x \le 1 \\ 0 & Otherwise \end{cases}$$

Where c is a constant

Determine the

(i) Value of c

(ii) Mean of x

(iii) Minimum capacity of the petrol tank if the probability that it will be exhausted in a given week is not to exceed 0.02.

4. Metal bars produced in a factory have masses that are normally distributed with mean μ and standard deviationσ. Given that 9.4% have a mass less than 45kg and 7% have a mass above 75kg.

(i) Evaluate the values of μ and σ and

(ii) Find the probability that a mass of a metal bar selected at random will be less tan 40kg.

5. Define statistics
6. Differentiate between mean, mode and median
7. Perform data collection and organization techniques
8. Perform interpretations analysis and presentation of data in appropriate format
9. Evaluate mean median mode and standard deviation obtained from the given data
10. Describe laws of probability and perform appropriate calculations based on the laws
11. In statistic we consider quantities that are varied. TRUE OR FALSE?
12. Measures of central tendencies is a single value which used to represent entire set of data. TRUE OR FALSE?

2.3.12. Learning Outcome No. 11. Solve Ordinary differential equations

2.3.12.1 Learning Activities

Learning Outcome No. 11. Apply Fourier series	
Learning Activities	Special Instructions
• Obtain Fourier coefficients using Fourier series • Obtain Fourier series for a periodic function of period 2π • Obtain Fourier series for a periodic function of period T • Obtain Fourier series for odd and even functions using Fourier series techniques • Obtain half-range Sine and Cosine series	• Harmonic analysis

2.3.12.2. Information sheet No 2/Lo11

Problem involving various forms of oscillations are common in field of modern technology and Fourier series enable us to represent a periodic function as an infinite trigonometrical series in sine and cosine series.

Periodic functions

A function $f(x)$ is said to be periodic if its function values repeat at regular intervals of the independent variable. The regular interval between repetitions is the period of the oscillations.

For more examples refer to Further Engineering Mathematics by K.A Stround on page 826.

Fourier series periodic functions of period 2π

We define Fourier series in the form

$$f(x) = \frac{1}{2}a_0 + a_1cosx + a_2cos2x + a_3cos3x \pm - - - +$$

$$+b_1sinx + b_2sin2x + b_3sin3x \pm - - -$$

This is written in the form

$$f(x) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_nCosnx + b_nsinnx)$$

Where n is a positive integer.

Fourier coefficients, $a_0, a_n, and\ b_n$ are given by

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cosnxdx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sinnxdx$$

For worked examples on how to determine the Fourier series to represent a periodic function of period 2π refer to Further *Engineering Mathematics by K.A Stround on page 842 to 876.*

Functions with periods other than 2π

If $y = f(x)$ is defined in the range $\frac{-T}{2}$ to $\frac{-T}{2}$ i.e has a period T, we can convert this to an interval of 2π by changing the units of the independent variable.

In many practical cases involving physical oscillations, the independent variable is time (t) and the periodic interval is normally denoted by T

i.e $f(t) = f(t + T)$

The Fourier series to represent the function can be expressed a

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n Cosnwt + b_n sinwt)$$

With the new variable, the Fourier coefficients becomes

$$f(t) = \frac{1}{2}a_0 + \sum_{n=1}^{\infty}(a_n Cosnwt + b_n sinwt)$$

$$a_0 = \frac{2}{T}\int_0^T f(t)dt$$

$$a_n = \frac{2}{T}\int_0^T f(t)cosntdt$$

$$b_n = \frac{2}{T}\int_0^T f(T)sinwtdt$$

For worked examples on how to determine Fourier series for a periodic function of period T, refer to *Further Engineering Mathematics by K.A Stround on page 879 to 886.*

Odd and even functions

For information about odd and even functions refer to Further Engineering Mathematics by K.A Stround on page 858. Also find worked out examples on odd and even functions.

Half-range series

Sometimes a function of period 2π is defined over the range 0 to π, instead of the normal $-\pi\ to\ \pi$ or 0 to 2 π. In this case we make an assumption on how a function behaves between $x = -\pi$ to $x = 0$, and the resulting Fourier series will therefore apply only to $f(x)$ between $x = 0$ and $x = \pi$ for which it is defined. For this reason, such series are called half range series. For more information, worked example and exercises refer to Further Engineering Mathematics by K.A Stround on page 868.

2.3.12.3. Self-Assessment

1. Describe Ordinary differential equations
2. Explain Fourier series

3. Perform calculations on Fourier series for a periodic function of period 2π and T
4. Perform calculations on Fourier series for odd and even functions using Fourier series techniques
5. A function $f(x)$ is said to be periodic if its function values repeat at regular intervals of the independent variable. TRUE OR FALSE?
6. If $y = f(x)$ is defined in the range $\frac{-T}{2}$ to $\frac{-T}{2}$ i.e has a period T. TRUE OR FALSE?
7. Determine half-range Sine and Cosine series
8. Determine the Fourier served for the function defined by

$$f(x) = \begin{cases} 1 - x & -\pi \leq x < 0 \\ 1 + x & 0 < x \leq \pi \\ & f(x + 2\pi) \end{cases}$$

Find the half-range Cosine series for the function defined by

$$f(t) = \begin{cases} 4 - t & 0 < t < 4 \\ & f(t + 8) \end{cases}$$

2.3.12.4. Tool, equipment

- Calculator

2.3.12.5 References

1. Further Engineering Mathematics by K.A Stround
2. Mathematics for Engineers by Dass

2.3.13. Learning Outcome No. 12. Apply Vector theory

2.3.13.1. Learning Activities

Learning Activities No. 12. Apply Vector theory	
Learning Activities	Special Instructions
• Calculate vector algebra dot and cross products using vector theory • Determine the gradient, divergence and curl. • Perform vector calculation using Greens theorem • Perform vector calculations using Stroke's theorem • Determine conservative vector fields, line and surface integrals using Gauss's theorem	

2.3.13.2. Information sheet No. 2/LO12

Physical quantities can be divided into two main groups, scalar quantities and vector quantities.

A Scalar quantity is one that is defined completely by a single number with appropriate units e.g Lengths, area, volume, mass, time etc.

A Vector quantity is defined completely when we know not only its magnitude but also the direction in which it operates, e.g. force, velocity, acceleration etc.

Refer to *Engineering mathematics by K. A Stroud* to learn more on components of 0 Vector in terms of unit Vectors on page 368.

Dot and cross product of vectors. The Scalar product of two vectors is denoted by $ā.\bar{b}$(sometimes called the 'dot product'.

The dot product of two vectors is defined as $a.b = |a||b|cos\theta$ where θ is the angle between a and b.

Refer to Technician mathematics 3 by J.O. Bird on page 297.

Examples

Solution

$\bar{a} = 2i + 3j + 5k$ And$\bar{b} = 4i + j + 6k,\ \bar{a}.\bar{b}$

$\bar{a}\ .\ \bar{b} = \ 2.4\ + 3.1\ + 5.6$

= 8 + 3+ 30

=41

A typical application of scalar products is that of determining the work done by a force when moving a body. The amount of work done is the product of the applied force and the distance moved in the direction of the applied force.

Example

Find the work done by a force F newtons acting at point A on a body, when A is displaced to point B, the coordinates of A and B being (3, 1, -2) and (4, -1, 0) metres respectively and when

$F = -i - 2j - k$ Newton's.

Solution

If a vector displacement from A to B is d, then the work done is F. d Newton Meters or joules. The position vector OA is $3i + j - 2k$ and OB is $4i - j$

$AB = d = OB - OA$

$= (4i - j) - (3i + j - 2k)$

$i - 2j + 2k.$

Work done $= F.d = (-1)/1) + (-2)(-2) + (-1)(2)$

$= -1 + 4 - 2$

$=1$ Nm or $joul$e

For more worked examples refer to Technician mathematics 3 by J. O. Bird.

Cross Product

The vector or Cross product of two vectors ā and $\bar{b}$ is C where the magnitude of C is $|ā||b|Sin\theta$ where θ the angle between is $\bar{a}$ and b.

For more information refer to Technician mathematics 3 by J.O Bird and Engineering mathematics by K. A Stroud.

Examples

$\bar{p} = 2i + 4j + 3k$ and $Q = i + 5j - 2k$ find $\bar{P}$ x $\bar{Q}$ $= \begin{vmatrix} i & j & k \\ 2 & 4 & 3 \\ 1 & 5 & -2 \end{vmatrix}$

$=i\begin{vmatrix} 4 & 3 \\ 5 & -2 \end{vmatrix} - j\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} + k\begin{vmatrix} 2 & 4 \\ 1 & 5 \end{vmatrix}$

$=-23i + 7j + 6k$

Typical applications of vector products are to moments and to angular velocity.

Refer to Technician mathematics. 3 by J.O Bird on page 308.

Vector field Theory

Refer to further Engineering mathematics by K.A Stroud to learn and also go through the worked examples and exercises on:

(i) Gradient
(ii) Divergence
(iii) Curl

Greens theorem

Learn how to perform vector calculations using Green's theorem by referring to further Engineer mathematics by KA. Stroud

Stoke's Theorem

Refer to further Engineer Mathematics by K.A Stroud to learn how to perform vector calculations using Stroke's theorem.

Gauss's Theorem

Refer to the some book to learn how to determine line and surface integrals using Gauss's theorem.

2.3.13.3 Self-Assessment

1) Differentiate between algebra dot and cross products in vector calculations
2) A Scalar quantity is one that is defined completely by a single number with appropriate units e.g Lengths, area, volume, mass, time. TRUE OR FALSE?
3) The vector or Cross product of two vectors ā and $\bar{b}$ is C where the magnitude of C is $|\bar{a}||b|Sin\theta$ where θ the angle between is $\bar{a}$ and b. TRUE OR FALSE?
4) Describe Greens theorem, Stroke's theorem and Gauss's theorem and their applications
5) Explain gradient, divergence and curl.
6) If $\bar{a}$ =$2i$ - $3j + 4k$ *and* $\bar{b}$ = i + $2j + 5k$ determine
   $(\acute{2})$ $\bar{a}$. $\bar{b}$
   $(2\acute{2})$ $\bar{a}$ x $\bar{b}$

Find the work done by a force F Newtons acting at a point A on a body, when A is displaced to point B ,the coordinates of A and B being (5, 2, -4) and (3,-1,1) meters respectively , and when $F = -2i - 3j - 2k$ Newton's.

2.3.13.4. Tools, Equipment

- Calculator

2.3.13.5 References

1. Further engineering mathematics by KA Stroud.
2. Mathematics for Engineers by Dass.
3. Technician mathematics book 3 by J.O. Bird.

2.3.14. Learning Outcome No.13 Apply Matrix

2.3.14.1 Learning Activities

Learning Activities No. 13. Apply Matrix	
Learning Activities	Special Instructions
• Apply matrix in calculating operation on matrices in addition and subtraction, multiplication	

2.3.14.2. Information sheet No. 2/ LO13

Introduction

A matrix is a set of real or complex numbers (or elements) arranged in rows and columns to form a rectangular array.

A matrix having M rows and N columns is called a $M \times N$ $(i.e\ M\ by\ N)$ matrix and is referred to as having order $M \times N$.

A matrix is indicated by writing the array with large square brackets e.g

$\begin{bmatrix} 5 & 7 & 2 \\ 6 & 3 & 8 \end{bmatrix}$ is a 2×3 matrix i.e $2\ by\ 3$ matrix where $5,7,2,6,3\ and\ 8$ are the elements of the matrix.

Operation on matrices

1. Addition and subtraction

Matrices can be added or subtracted if they are of the same order.

Example

Given matrix $A = \begin{bmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \\ 1 & -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 6 \\ 1 & -4 & 5 \end{bmatrix}$

$$A + B = \begin{bmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \\ 1 & -3 & 2 \end{bmatrix} + \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 6 \\ 1 & -4 & 5 \end{bmatrix} = \begin{bmatrix} (-1+3) & (2+2) & (3 \pm -1) \\ (3+-2) & (0+1) & (4+6) \\ (1+1) & (-3+-4) & (2+5) \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 4 & 4 \\ 1 & 1 & 10 \\ 2 & -7 & 7 \end{bmatrix}$$

2. Multiplication of Matrices

Example

Given matrix $A = \begin{bmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \\ 1 & -3 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 6 \\ 1 & -4 & 5 \end{bmatrix}$

$$A \times B = \begin{bmatrix} -1 & 2 & 5 \\ 3 & 0 & 4 \\ 1 & -3 & 2 \end{bmatrix} \begin{bmatrix} 3 & 2 & -1 \\ -2 & 1 & 6 \\ 1 & -4 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 3 + 2 \times -2 + 5 \div 1 & -1 \times 2 + 2 \times 1 + 6 \times -4 & -1 \times 1 + 2 \times 6 + 5 \times 5 \\ 3 \times 3 + 0 \times -2 + 4 \times 1 & 3 \times 2 - 0 \times 1 + 4 \times -4 & 3 \times -1 + 0 \times 6 + 4 \times 5 \\ 1 \times 3 + -3 \times -2 + 2 \times 1 & 1 \times 2 - 3 \times 1 + 2 \times -4 & 1 \times -1 + 6 \times -3 + 2 \times 5 \end{bmatrix}$$

$$= \begin{bmatrix} -3 - 4 + 5 & -2 + 2 - 20 & -1 + 12 + 25 \\ 9 + 0 + 4 & 6 + 0 - 16 & -3 + 0 + 20 \\ 3 + 6 + 2 & 1 - 3 - 8 & -1 - 1 + 10 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -20 & 36 \\ 13 & *10 & 17 \\ 11 & -10 & -9 \end{bmatrix}$$

Note that in matrices $AB \neq BA$

For more examples on matrices operations refer to Pure Mathematics I.

Determinant of a 3×3 matrix

Refer to engineering mathematics by K.A Stround and learn more on how to find the determinant of a 3×3 ("3 by 3") matrix.

Inverse of a 3×3 matrix

Example

To find the inverse of $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{bmatrix}$

Evaluate the determinant of A i.e $|A|$

a) For $A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 1 & 5 \\ 6 & 0 & 2 \end{bmatrix}$, $|A| = 1(2-0) - 2(8-30) + 3(0-6) = 28$

b) Now from the matrix of the cofactors

$$C = \begin{bmatrix} 2 & 22 & -6 \\ -4 & -16 & 12 \\ 7 & 7 & -7 \end{bmatrix}$$

c) Next we have to write down the transpose of "C" to find the adjoint of "A"

$$Adj\ A = C^T = \begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix}$$

d) Finally we divide the elements of adj A by the value of $|A|$ i.e 28 to get A^{-1} the inverse of A.

$$A^{-1} = \begin{bmatrix} \frac{2}{28} & \frac{-4}{28} & \frac{7}{28} \\ \frac{22}{28} & \frac{-16}{28} & \frac{12}{28} \\ \frac{-6}{28} & \frac{7}{28} & \frac{-7}{28} \end{bmatrix} = \frac{1}{28} \begin{bmatrix} 2 & -4 & 7 \\ 22 & -16 & 7 \\ -6 & 12 & -7 \end{bmatrix}$$

Solution of linear equation in three unknowns

Solve the set of equations

$$x_1 + 2x_2 + x_3 = 4$$
$$3x_1 - 4x_2 - 2x_3 = 2$$
$$5x_1 + 3x_2 + 5x_2 = -1$$

First write the set of equations in matrix form

$$\begin{bmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 6 & 3 & 5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 4 \\ 2 \\ 1 \end{bmatrix}$$

Next step is to find the inverse of A where A is the square matrix on the left hand side.

$$|A| = \begin{vmatrix} 1 & 2 & 1 \\ 3 & -4 & -2 \\ 5 & 3 & 5 \end{vmatrix} = -14 - 50 + 29 = 29 - 64 = -35$$

Therefore $|A| = -35$

Matrix of co-factors $C = \begin{bmatrix} -14 & -25 & 29 \\ -7 & 0 & 7 \\ 0 & 5 & -10 \end{bmatrix}$

The matrix of the $Adj\ A = C^T = \begin{bmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{bmatrix}$

Now $|A| = -35$, therefore $A^{-1} = \frac{adj\ A}{|A|} = \frac{1}{35}\begin{bmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{bmatrix}$

$$\therefore \quad x = A^{-1}.b = \frac{1}{35}\begin{bmatrix} -14 & -7 & 0 \\ -25 & 0 & 5 \\ 29 & 7 & -10 \end{bmatrix}\begin{bmatrix} 4 \\ 2 \\ -1 \end{bmatrix}$$

Finally $x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ -4 \end{bmatrix}$

$$\therefore x_1 = 2, x_2 = 3, x_3 = -4$$

Eigen values and Eigen vectors

In many applications of matrices to technological problems involving coupled oscillations and vibrations equations are of the form

$$A.x = \lambda x$$

Where A is a scale matrix and λ is a number. The values of λ are called he eigenvalues. Characteristic values or latent roots of the matrix A and the corresponding solution of the given equation $A.x = \lambda x$ are called the eigevectors or characteristics vector of A.

For more information refer to Engineering Mathematics by K.A Stroud.

2.3.14.3. Self- Assessment

1. Define a matrix
2. A matrix having M rows and N columns is called a $M \times N$ $(i.e\ M\ by\ N)$ matrix and is referred to as having order $M \times N$. TRUE OR FALSE?
3. $A.x = \lambda x$ are called the eigevectors or characteristics vector of A. TRUE OR FALSE?
4. Differentiate between Eigen values and Eigen vectors
5. Perform calculations using matrix
6. Determine a Determinant of a 3×3 matrix
7. Determine an Inverse of a 3×3 matrix
8. Solve the following set of linear equations by matrix method

   $x_1 + 3x_2 + 2x_3 = 3$

   $2x_1 - x_2 - 3x_3 = -8$

   $5x_1 + 2x_2 + x_2 = 9$

9. Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 5 & 1 \\ 2 & 0 & 6 \end{bmatrix}$

10. If $A.x = \lambda x$, $A = \begin{bmatrix} 2 & 2 & -2 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. Determine the eigenvalues of the matrix A and an eigenvector corresponding to each eigenvalue.

2.3.15. Learning Outcome No.14. Apply Numerical Methods

2.3.15.1. Learning Activities

Learning Activities No.14 Apply Numerical Methods	
Learning Activities	Special Instructions
• Apply numerical methods in solving engineering problems	

2.3.15.2. Information Sheet No. 2/ LO14

The limitation of analytical methods, have led engineers and scientist to evolve graphical and numerical methods. The graphical methods though simple give results to a low degree of accuracy. Numerical methods can, however, be derived which are more accurate.

Numerical methods are often of a repetitive nature. These consist of repeated execution of the same process where at each step the results of the proceeding step is used. This is known as iteration process and is repeated till the result is obtained to a derived degree of accuracy.

The numerical methods for the solution of algebra and transcendental equations include

1) Method of false solution/ regular false method
2) Newton-Raphson method.

Example

Find by Newton Raphson method the real roots of $xe^x - 2 = t$ correct to 3 decimal places.

Solution

Let $f(x) = xe^x - 2$ and $f(1) = e - 2 = 0.7183$

So a root lies between 0 and 1. Its near to 1 let's take $x_0 = 1$

Also $f'(x) = xe^x + e^x$ and $f'(1) = e + e = 5.4366$

Therefore by Newton-Raphson rule, the first approximation x_1 is given by

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)} = 1 - \frac{0.7183}{5.4366} = 0.8679$$

Therefore $f(x_1) = 0.0672, \quad f'(x_1) = 4.4491$

Thus the first approximation x_2 is given by

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 0.8679 - \frac{0.0672}{4.4491} = 0.8528$$

Hence the required root is 0.853 correct to 3 dp

Finite differences and interpolation

Suppose we are given the following values for $y = f(x)$ for a set of values of x:

$$x: x_0 \;\; x_1 \;\; x_2 \;\; \dots . x_n$$
$$y: y_0 \;\; y_1 \;\; y_2 \;\; \dots . y_n$$

Then the process of finding any value of y corresponding to any value of $x = x_i$ between x_0 and x_n is called interpolation.

The interpolation is the technique of estimation the value of a function for any value of the intermediate value of the independent variable while the process of computing the value of the function outside the given range is called extrapolation.

Example

The table gives the distance in nautical miles of the visible horizon for the given height in feet above the earth's space.

Height (x)	100	150	200	250	300	350	400
Distance (y)	10.63	13.03	15.04	16.81	18.42	19.90	21.27

Find values of y when x =2.8 feet and 410 $feet$

The difference table is as given below

x	y	Δ	Δ^2	Δ^3	Δ^4
100	10.63				
		2.40			
150	13.03		-0.39		
		2.01		0.15	
200	15.04		-0.24		-0.07
		1.77		0.08	
250	16.81		-0.16		-0.05
		1.61		0.03	
300	18.42		-0.13		-0.01
		1.48		0.02	
350	19.90		-0.11		
		1.37			
400	21.27				

i) If we take $x_0 = 200$, then $y_0 = 15.04$, $\Delta_{y_0} = 1.77$ $\Delta^2{}_{y_0} = -0.16$ $\Delta^3{}_{y_0} = 0.03$ etc

Since $x = 218$ and $h = 50 \therefore p = \frac{x - x_0}{h} = \frac{18}{50} = 0.36$

Using Newton Raphson forward interpolation formula, we get

$$y_{218} = y_0 + p\Delta_{y_0} + \frac{p(p-1)\Delta^2{}_{y_0}}{1.2} + \frac{p(p-1)(p-2)}{1.2.3} \quad \Delta^3{}_{y_0} + \cdots$$

$$f(218) = 15.04 + 036(1.77) + \frac{0.36(-0.64)}{2}(-0.16)$$

$$+ \frac{0.36(0.64)(-1.64)(0.03)}{6}(0.03) + \cdots$$

$$= 15.04 + 0.637 + 0.018 + 0.001 + \cdots. = 15.696\,nautical\ miles$$

ii) Since $x = 410$ is near the end of the table, we use Newton backward interpolation formula.

Therefore taking $x_n = 400p = \frac{x-x_n}{h} = \frac{10}{50} = 0.2$

Using the backward differences

$y_n = 21.27, \nabla_{y_n} = 1.37 \quad \nabla^2{}_{y_n} = 0.11 \quad \nabla^3{}_{y_n} = 0.02$

Newton backward difference gives

$$y_{410} = y_{405} + p\Delta_{y_{400}} + \frac{p(p+1)\nabla^2{}_{y_{400}}}{1.2} + \frac{p(p+1)(p+2)}{1.2.3} \quad \nabla^3{}_{y_{400}} + \cdots$$

$$21.27 + 0.2(1.37) + \frac{0.2(1.2)}{2}(-0.11) + \cdots . = 21.53\ nautical\ miles$$

Refer to *Higher Engineering Mathematics by Dr B.S* and learn forward and backward interpolation formulae

2.3.15.3 Self-Assessment

1. Describe numerical methods
2. The graphical methods though simple give results to a low degree of accuracy. Numerical methods can, however, be derived which are more accurate. TRUE OR FALSE
3. The process of computing the value of the function outside the given range is called interpolation. TRUE OR FALSE
4. Differentiate between Method of false solution/ regular false method and Newton-Raphson method.
5. State the applications of numerical methods
6. Using Newton-Raphson forward formula find the values of $f(1.6)$, if

X	1	1.4	1.8	2.2
$f(x)$	3.49	4.82	5.96	6.5

7. State Newton interpolation formula and use it to calculate the value of $\exp(1.85)$ given the following table

X	1.7	1.8	1.9	2.0	2.1	2.2	2.3
$f(x)$	5.474	6.000	6.686	7.389	8.166	9.025	9.914